Diversion Ratios and Market Elasticity: Some Useful Formulas [1]

Last Updated: October 2012

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This memo derives several useful formulas describing the relationship between "diversion ratios" and the "market elasticity" of demand.

1. Definitions

There are N differentiated products. The demand for product k is given by:

$$q_k = D_k(p_1,\ldots,p_N) \qquad , \qquad k=1,\ldots,N \tag{1}$$

where q_k is the quantity demanded, D_k is the demand function, and p_j is the price of product j ($j=1,\ldots,N$). The own-price elasticity of demand for product k is given by:

$$\varepsilon_{kk} = \frac{\partial D_k}{\partial p_k}\frac{p_k}{q_k} \tag{2}$$

The total demand for the N products is given by:

$$q = \sum_{k=1}^{N} D_k(p_1,\ldots,p_N) \tag{3}$$

and the aggregate elasticity of total demand (with respect to a proportional increase in all the prices) is given by:

$$E = \frac{t}{q}\frac{d}{dt}\sum_{k=1}^{N} D_k(tp_1,\ldots,tp_N)\Bigg|_{t=1} = \sum_{k=1}^{N}\sum_{j=1}^{N}\frac{\partial D_k}{\partial p_j}\frac{p_j}{q} \tag{4}$$

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[1] This note has been revised repeatedly over the years. This document is available at http://crai.com/expert/serge-x-moresi.

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This definition of “market elasticity” is based on total volume and produces an elasticity measure that generally is dependent on the units of measurement chosen for each product. In some cases, therefore, one might prefer to use a definition based on total revenues (or total spending):

$$r = \sum_{k=1}^{N} p_k D_k(p_1,\ldots,p_N) \tag{3*}$$

and define the aggregate elasticity of total demand as being equal to the elasticity of total revenue minus 1:

$$E^* = \frac{t}{r}\frac{d}{dt}\sum_{k=1}^{N} tp_k D_k(tp_1,\ldots,tp_N)\Big|_{t=1} - 1 = \sum_{k=1}^{N}\sum_{j=1}^{N} p_k \frac{\partial D_k}{\partial p_j}\frac{p_j}{r} \tag{4*}$$

This alternative definition is useful when firms sell multiple products and/or there is no natural way to choose units of measurement.

The diversion ratio from product j to product k is given by:

$$\delta_{jk} = -\frac{\partial D_k / \partial p_j}{\partial D_j / \partial p_j} \tag{5}$$

This measure also depends on measurement units, and thus one might prefer to use:

$$\delta_{jk}^{*} = -\frac{p_k \partial D_k / \partial p_j}{p_j \partial D_j / \partial p_j} \tag{5*}$$

The retention ratio (or market recapture rate) for product j is the total diversion ratio from product j to all the other products:

$$R_j = \sum_{k \neq j} \delta_{jk} \tag{6}$$

and the corresponding unit-independent measure is:

$$R^*_j = \sum_{k \neq j} \delta^*_{jk} \tag{6*}$$

The diversion ratio from product j to "outside goods" is defined to be equal to $1 - R_j$ (or $1 - R^*_j$). In general, different products may have different retention ratios and thus different diversion ratios to outside goods.

2. Relationship between elasticities and retention ratios

Equation (4) can be rewritten as:

$$E = \sum_{j=1}^{N} \left(\frac{p_j}{q} \sum_{k=1}^{N} \frac{\partial D_k}{\partial p_j} \right) \tag{7}$$

(Simply reverse the order of summation in Equation (4) and then factorize p_j / q.) Then, using Equation (5), Equation (7) can be rewritten as:

$$E = \sum_{j=1}^{N} \left(\frac{p_j}{q} \frac{\partial D_j}{\partial p_j} \left[1 - \sum_{k \neq j} \delta_{jk} \right] \right) \tag{8}$$

Using Equations (2) and (6), Equation (8) can be rewritten as:

$$E = \sum_{j=1}^{N} s_j \varepsilon_{jj} (1 - R_j) \tag{9}$$

where $s_j = q_j / q$ is the quantity share of product j.

Equation (9) shows the relationship between the aggregate elasticity of total demand and the own-price elasticities, the retention ratios and the quantity shares of the individual products. This relationship also can be expressed in terms of unit-free, revenue-based variables as follows:

$$E^* = \sum_{j=1}^{N} s_j^* \varepsilon_{jj} (1 - R_j)^* \qquad (9^*)$$

where $s_j^* = p_j q_j / r$ is the revenue share of product j.

3. Symmetric retention ratios

If one assumes symmetric retention ratios (i.e., $R_j = R$), then Equation (9) implies:

$$E = (1 - R) \sum_{j=1}^{N} s_j \varepsilon_{jj} \qquad (10)$$

Assuming that each product is sold by a different firm, the profit margin of each product is equal to (in Bertrand-Nash equilibrium):

$$m_j = 1/\varepsilon_{jj} \qquad (11)$$

(This is known as the "inverse elasticity rule".) Using Equation (11) to substitute for ε_{jj} in Equation (10), one obtains the "RIME formula":

$$R = 1 - \bar{M}E \qquad (12)$$

where $\bar{M}$ is the volume-weighted harmonic average margin:

$$\bar{M} = \frac{1}{\sum_{j=1}^{N} s_j / m_j} \qquad (13)$$

In terms of unit-independent variables, we have:

$$R^* = 1 - \bar{M}E^* \qquad (12^*)$$

where $\bar{M}$ is the volume-weighted harmonic average margin:

$$\bar{M} = \frac{1}{\sum_{j=1}^{N} s_j / m_j} \tag{13}$$

In the case with symmetric margins, we have $m_j = M$ and therefore the RIME formula in Equation (12) applies with $\bar{M} = M$.