

Inverse Floaters and the Income Stability of a Debt Securities Investment Portfolio

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Inverse floaters are generally thought to be volatile securities. And, standing alone, they may in fact be volatile. If they are used properly in a portfolio context, however, they can serve as hedging vehicles for a buy-to-hold investor concerned with reinvestment income and thereby reduce the income volatility of a debt securities portfolio. This result occurs because, during periods of low interest rates, high coupon payments from inverse floaters offset the lower reinvestment rate, and vice versa. The presence of inverse floaters thereby creates a natural hedge against reinvestment risk.

Using a two-factor interest rate model and Monte Carlo simulations, this study compares the expected income and volatility of seven different investment portfolios: 3-month T-bills, 5-year T-notes, 5-year zero-coupon bonds, 5-year inverse floaters, T-notes plus inverse floaters, and T-notes plus inverse floaters plus repurchase agreements in varying amounts. The simulated results are reinforced by empirical evidence. The simulations and empirical analyses offer several interesting findings. First, under certain investment objectives and risk profiles, inverse floaters can play an economic role in augmenting the level of income as well as stabilizing the income volatility of the portfolio. Second, contrary to the common perception that T-bills constitute a riskless investment, investors could be subject to a substantial degree of reinvestment risk if their investment horizons are longer than the maturity of the invested T-bills.

The objective of this paper is not to argue that a portfolio consisting purely of inverse floaters outperforms other investment strategies, because it does not; rather, the paper seeks to dispel the bad publicity that arose following several municipal financial crises concerning the suitability of these instruments.

Why Are Inverse Floaters Volatile?

Floating rate instruments are a class of securities that feature interest payments, or coupons, which change at specified reset intervals according to a predetermined formula.

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On scheduled reset dates, the coupon rates are adjusted (either fully or partially) to the prevailing market rates, a process commonly known as re-pricing. This re-pricing has the effect of avoiding large price swings in the security as the general level of interest rates fluctuates over time. The more frequent the reset dates and the less constrained the adjustments are to move fully with current interest rate levels, the closer to par value the security will trade.¹ Therefore, as a general rule, floating rate securities tend to exhibit less price volatility than their fixed rate counterparts, *ceteris paribus*.

Inverse floating rate instruments, also commonly known as inverse floaters, are often created out of a fixed income instrument whose payments are divided into two parts and sold separately. A floating rate instrument is sold to one group of investors, while an inverse floater is sold to the other group.² Unlike a simple floater, the coupon rate of an inverse floater is designed to move in a direction opposite to that of some reference or market interest rate. Thus, the coupon rate increases as interest rates decrease. This property permits investors to benefit from a falling rate environment. However, this characteristic makes an inverse floater different from a simple floater in that there is no longer any assurance that a coupon reset would bring the value of an inverse floater back towards par. In other words, the interest rate risk related to the fixed rate instrument that gives rise to the two components (i.e., floater and inverse floater) is primarily absorbed by the inverse floater. Consequently, the interest rate risk inherent in an inverse floater is not only higher than that of a simple floater, but on a relative, or percentage basis, it is even higher than that of a fixed rate note.

Inverse Floaters Could Be Attractive to Certain Investment Portfolios

As discussed above, inverse floaters can experience significant price volatility, doing very well when interest rates drop and very poorly when interest rates rise. While it has been well recognized that their coupon rates fluctuate inversely with market rates, some have not recognized that this feature imparts a high duration to the instruments. In part for this reason, inverse floaters have received bad publicity regarding their suitability as an investment class. Nevertheless, inverse floaters do provide investors with a unique risk-return profile that can fit very well in the context of an overall portfolio.

Holding Securities to Maturity

In general, prices of fixed income securities move inversely to changes in interest rates. For investors who sell a fixed income security before its maturity date, an increase

¹ In this statement, we are ignoring basis risk occasioned by credit and/or liquidity risk. Of course, some floating rate securities feature significant amounts of these risks, and may not necessarily re-price close to par at reset dates.

² Not all floaters are created by dividing fixed-rate collateral into a floater and an inverse. Some are created as structured notes where the inverse is created without a corresponding floater. This is done in the muni, agency, and corporate market using swaps and caps.

in interest rates may mean the realization of a capital loss. However, investors who have both the intent and financial resources to hold securities to maturity would find these interim price changes to be of less concern. As long as investors can ride out the short-term market volatility, in the absence of default risk, they will receive the full principal back on the maturity date.

Maintaining a Steady Income Stream

A substantial portion of the return from investing in fixed income securities comes from the periodic coupon income and the reinvestment of these interim cash flows. Variability in the interest rates at which these payments can be reinvested constitutes reinvestment risk.³ The degree of exposure to reinvestment risk depends in large part on the investor's time horizon. For example, an investor who does not need access to the invested funds for more than five years will experience significant exposure to reinvestment risk by investing in 3-month T-bills. During a five-year period, this investor would need to reinvest the entire portfolio 19 times; therefore, in a falling interest rate environment, there can be significant earnings erosion.

For a hold-to-maturity investor who expects a certain level of income from investments in order to meet operating expenses, or for an endowment, trust, or other fund restricted to maintaining the corpus and spending only the portfolio income, the risk of inadequate income is important. Allocating a portion of the investment funds to inverse floaters is a viable way to hedge against that risk. In a falling rate scenario, the increase in coupon income from the inverse floaters will offset the reduction in reinvestment income. Of course, in a rising rate environment, the inverse floaters will generate lower coupon income and if interest rates rise sufficiently high, the interest income can fall to zero.⁴ Investors will benefit, however, from rising interest rates through reinvestment of coupon income. Unless there is a concomitant and equal rise in inflation accompanying rising interest rates, hold-to-maturity investors will ultimately be better off in terms of the income they receive from their investments, in spite of the fact that rising rates would have reduced the current market value of their investments.⁵

³To some degree, reinvestment risk and market risk can be partially offset. Market risk addresses the exposure of rising interest rates and their effect on security prices, while reinvestment risk points to the possibility of lower interest rates and their effect on the earnings of the reinvested coupon stream.

⁴Inverse floaters typically contain interest rate floors that prevent their coupons from going below a specified level, normally, 0%, no matter how large an adverse interest rate move might occur. One advantage of such a feature is that if the worst case is realized and the coupon does go to zero, the inverse floater begins to approximate a zero-coupon note. (Because the interest rate may yet fall again, causing the coupon on the inverse floater to become positive, the instrument will never be identical to a zero-coupon note.) If held to maturity, the inverse floater will still enable the investor to redeem the full principal in spite of the adverse interest rate movement.

⁵See Levine [1996].

Comparative Income Volatility Analyses

In order to examine the income stabilization contributions of inverse floaters to a portfolio of fixed income securities, the income volatility characteristics of four hypothetical portfolios are compared using Monte Carlo simulations. Five thousand random paths of interest rate movements over five years are generated using the Schaefer and Schwartz two-factor interest rate model (“SS model”).⁶ Appendix 1 contains a brief description of the SS model. It also lays out the methodology and assumptions used to calibrate the relevant model parameters using 10-year historical data.

The simulated portfolios, each of which has an initial investment of \$100, are constructed as follows:

- ◆ Portfolio 1: The entire \$100 is invested in a 3-month T-bill that is rolled over at the prevailing 3-month T-bill rate.⁷
- ◆ Portfolio 2: The entire \$100 is invested in a 5-year Treasury Note and the coupon payments are reinvested at the prevailing 6-month T-bill rate.
- ◆ Portfolio 3: The entire \$100 is invested in a 5-year inverse floater issued by government agencies.⁸ The coupon structure of this note is [12.98% minus 6-month LIBOR].⁹ The semi-annual coupon payments are reinvested at the prevailing 6-month LIBOR.
- ◆ Portfolio 4: This portfolio has a mix of 75% 5-year T-notes and 25% inverse floaters. Out of the \$100, \$75 is invested in the 5-year T-note and the coupon payments are reinvested at the prevailing 6-month T-bill rate. The remaining \$25 is invested in an inverse floater and the coupon payments are reinvested in a 6-month LIBOR instrument.

Each simulation entailed the use of a common set of 5,000 random interest rate paths that were generated based on the SS model. Both the 5-year T-note and inverse

⁶See Schaefer and Schwartz [1984].

⁷ All interest rate data used in this paper are converted to a bond equivalent yield basis.

⁸ Government Sponsored Enterprises (GSEs or agencies), such as the Federal National Mortgage Association, Government National Mortgage Association, Federal Home Loan Bank, Federal Home Loan Mortgage Corporation, and Student Loan Marketing Association, have the highest possible credit status below the U.S. Government. U.S. Treasuries and agencies are usually presumed to have virtually no default risk.

⁹ Based on the then prevailing interest rates in the market on December 31, 1991, the coupon is structured to ensure that the inverse floater is valued at par. Using data from Bloomberg, the 5-year swap rate was 6.49% on December 31. Also, for simplicity reasons, the inverse floater has no implicit caps and floors.

floater are valued at par at the beginning of the 5-year simulation period. The use of the 75%-25% asset allocation is merely arbitrary. This asset allocation figure can vary according to one's investment horizon, objectives, risk tolerance and the leverage embedded in the coupon structure of inverse floaters.¹⁰

Table 1 summarizes the simulation results: the average amount of income expected to be accumulated at the end of five years as well as the corresponding expected volatility measured in terms of standard deviation. The results indicate that an investment portfolio consisting of 100% T-bills has the worst performance relative to other portfolios. Not only does it have the lowest average expected income, but it also shows the highest income volatility. The income of Portfolio 1 (100% T-bill) is almost 6 times more volatile than that of Portfolio 2 (T-note), and 21 times more volatile than Portfolio 4 (T-notes plus inverse floaters).¹¹ Figure 1 depicts the dispersion of the expected income of the five portfolios. As shown, the T-bill portfolio exhibits the highest income volatility and the expected income varies over a wide range.

The Impact of Leverage on Portfolio Performance

Greater expected returns are usually accompanied by higher risk. In order to examine the impact of leverage on portfolio performance, the simulation was repeated with Portfolio 4 leveraged by a factor of 1.5 using 6-month repurchase agreements.¹² The \$50 proceeds received from reversing the securities in the portfolio are then used to purchase additional 5-year T-notes and 5-year inverse floaters in the same 75%-25% mix. The interest rate of the repo is assumed to be the 6-month LIBOR.

Figure 2 compares the income dispersion of this leveraged portfolio (Portfolio 5) with the T-bill portfolio. Injecting leverage into Portfolio 4 does generate a higher cumulative expected income, from \$39.67 to \$43.22, at the end of the 5-year simulation period. As expected, the income is also more volatile as the standard deviation increases

¹⁰ Inverse floaters with a simple coupon structure are used in the simulation analyses. However, inverse floaters can vary in their coupon structure to reflect different risk and return characteristics. The effective "leverage" of some inverse floaters can be several times than that of others.

¹¹ Portfolio 4 is expected to have low income volatility because its initial duration comes close to matching the investment horizon. Because of the significant asset/liability duration gap (i.e., 3-month maturity versus 5-year investment horizon), the T-bill portfolio demonstrates the highest income volatility.

¹² The terms repurchase agreement (repo) and reverse repurchase agreement (reverse repo) refer to transactions in which market participants acquire immediately available funds by selling securities, and simultaneously agreeing to repurchase the same or similar securities after a specified time period at a given price. The difference between the sales price and the repurchase typically reflects the interest cost for that particular time frame. The terminology is typically approached from the dealer's point of view. When a dealer first buys a security from an investor and agrees to sell it back later, it is executing a reverse repo transaction. A repo is just the opposite. In this paper, we will not make that distinction.

from \$0.39 to \$4.51. Nevertheless, the income volatility of the leveraged Portfolio is still less than that of the T-bill portfolio.

Figure 3 illustrates the income dispersion when Portfolio 4 is leveraged by a factor of 1.94 instead of 1.5 (Portfolio 6). It is at the leverage ratio of 1.94 that the income volatility of Portfolio 6 is the same as the T-bill Portfolio. The standard deviations of the two portfolios are exactly the same (\$8.23). Yet, this leveraged portfolio generates higher expected average income than that of T-bill portfolio (\$46.34 versus \$32.57, 42% higher).

An Efficiency Analysis of Investments under Uncertainty

In previous sections, means and standard deviations were used to examine the income and risk characteristics of different investment strategies. While these two moments of the return distribution are sufficient to order the desirability of competing portfolios for investors characterized by quadratic utility, for most investors this will be inadequate. Stochastic dominance efficiency criteria, which take into account all moments of a return distribution, are applicable to a much broader spectrum of investors.¹³ First degree stochastic dominance is applicable to all investors who prefer more wealth to less wealth, while second degree dominance is applicable to the class of all risk averse investors. We applied both criteria to screen the alternative portfolios.

Figure 4 depicts the cumulative probability distributions (CPDs) of Portfolios 1 through 5. First degree stochastic dominance requires that the preferred portfolio has at least as high a probability of exceeding any particular level of return than other portfolios. This dominance can be demonstrated by CPDs that do not cross over one another, with the dominant portfolio exhibiting the CPD furthest to the right. Under the first-order stochastic dominance, no investment strategies dominate the T-bill portfolio.¹⁴ Figure 5 shows the cumulative areas under the CPD for all the five portfolios. It appears that Portfolio 2 (T-note), Portfolio 5 (T-note plus inverse floater) and Portfolio 6 (leveraged) second-order stochastically dominate Portfolio 1 (T-bills).¹⁵

¹³ See Ingersoll [1987, pp. 71-73, 122-124, 137-139].

¹⁴ One portfolio (or asset) dominates another if it always outperforms the other. To stochastically dominate a second portfolio, the first portfolio need not always outperform, but the first portfolio's probability of exceeding any given level of return must be higher than that of the second portfolio. Under the scenario of high interest rates, the four other portfolios will under-perform relative to the T-bill portfolio. That violates the first-order stochastic dominance rule that requires the out-performance to be true for all possible ranges of interest rate movements.

¹⁵ Under the second-order stochastic dominance rule, Portfolio B dominates Portfolio A for all risk averse investors if and only if the area under the cumulative probability distribution of A exceeds the area under the cumulative distribution of B.

The inferences drawn from applying the stochastic dominance rules reinforced the findings in the earlier sections. With the exception of Portfolio 4 (inverse floaters), over an investment horizon of five years, all the other three portfolios are preferred to the 3-month T-bill portfolio.

Investment Environment Favorable for Inverse Floaters

As demonstrated in Table 1 and Figure 1, the inclusion of inverse floaters in the T-note portfolio increases the expected income but lowers the income volatility under the 5-year investment horizon. While the reduction in income volatility is to be expected, the higher average income should not always be expected. Investors should consider including inverse floaters in the portfolio only under favorable market conditions like the steep yield curve environment in the early 1990s. The starting date of the simulation is December 31, 1991, a date characterized by such a steep yield curve.

In general, inverse floaters are designed to perform especially well during periods of stable or declining interest rates with steep yield curves. They will also perform well during periods of slightly rising interest rates. Only sharply rising interest rates would stress a portfolio operating under this strategy. Even then, the investments would be structured such that the principal would be preserved as long as they were held to maturity. As such, the market witnessed a proliferation of inverse floaters in the investment community in early 1990s.

Empirical Evidence: Backtesting

The inferences discussed in the earlier sections concerning the income volatility of different investment strategies are drawn from the simulated results using the SS model under the market conditions at the end of 1991. This section presents empirical evidence of the comparative performance of these strategies based on actual interest data for the period from February 1, 1973 through April 30, 1997.

Two different investment approaches are examined. The first one can be described as a naïve strategy by simply purchasing inverse floaters without taking into the consideration the market conditions. The second approach includes inverse floaters in the T-note portfolio only when initial market conditions are favorable — a steep yield curve with a wide spread between the short rate and the long rate to provide additional cushion.

Without Market Timing

Assume that at the beginning of each month, starting from February 1, 1973 until April 1, 1992, \$100 is invested in each of the five portfolios for five years. This gives rise to a sample of 231 observations. The assumptions underlying the original simulation

analyses still apply, with the exception that the income generated by each of the five portfolios is calculated based on interest rates that prevailed during the relevant time frame.¹⁶ In other words, after the initial purchase of \$100 worth of 3-month T-bills, Portfolio 1 is rolled over 19 times at the then prevailing 3-month T-bill rate until the end of 5-year investment horizon. The semi-annual coupon income of the 5-year T-note is reinvested in 6-month T-bills. With respect to the inverse floater, the constant component of the coupon structure is fixed at the initial purchase date to make the purchase price equal to par value and the actual coupon rate is then reset every six months based on the prevailing 6-month T-bill rate.

Table 2 provides the summary statistics (minimum, maximum, mean and standard deviation) of these 231 observations. Figure 6 depicts the income generated by each of the three portfolios at the end of each of the 5-year investment programs (i.e., February 1, 1978 through April 1, 1997). As shown, the T-bill portfolio outperforms the other two portfolios during two distinct periods — January 1981 through July 1985 and September 1991 through February 1992. This performance is due to the rising interest rate environment and inverted yield curves that characterized the marketplace during much of the 1977-81 period, and the rising yields and flattening yield curve that characterized the 1988-89 period. As discussed earlier, the T-bill portfolio benefits in a rising interest rate environment because coupons are reinvested at higher rates.

With Market Timing

The second investment approach assumes that inverse floaters will be included only when the spread between the 3-month T-bill and the 5-year Treasury note exceeds a critical level. Figure 7 graphs the spreads between these two rates for the period from February 1973 through April 1997 while Table 3 tabulates the spreads into different intervals. As shown in Table 3, spreads exceeded 200 basis points 28 percent of the time during this period, and exceeded 100 basis points 63 percent of the time.

Table 4 compares the differences in income between the T-bill portfolio and the T-note portfolio with the addition of inverse floaters under various critical spread levels. Column 1 lists the different critical levels, ranging from 2% to 2.6% between the 5-year CMT and the 3-month T-bill rates. The “None” level is included as a reference to the first naïve investment approach. Column 2 states the number of months that the spread exceeds a particular critical level. Columns 3 through 5 provide the minimum, maximum and average value of the income differences between the two portfolios at a particular spread level. Columns 6 and 7 show the number of times and the percentage that the T-bill portfolio outperforms the T-note plus inverse floater portfolio.

¹⁶ Five-year swap rate data are only available from Bloomberg beginning April 1989. Therefore, the coupon of the inverse floater is structured such that the constant component is based on the 5-year Treasury rate while the floating component is tied to the 6-month T-bill rate.

For example, during the February 1973 and April 1997 period (231 months), there are 63 months that the spread exceeds the 2% critical level. Of these 63 occurrences, if \$25 inverse floaters were purchased and added to the \$75 T-note portfolio to form the \$100 initial portfolio, on average, that portfolio would have generated \$17.42 more in income than the T-bill portfolio at the end of the 5-year investment horizon. However, the T-note plus inverse floater portfolio could have out-performed by as much as \$46.53 or under-performed by \$25.32.

Table 4 indicates that the higher the critical level, the higher is the average income that the T-note plus inverse floater portfolio will exceed the T-bill portfolio. The magnitude of the under-performance will also reduce. However, as shown in Column 2, that benefit comes at the cost of fewer occurrences. In other words, by selecting a higher spread level before establishing a position in inverse floaters, the investor obtains additional cushion to protect against adverse interest rate movements on the coupon structure. However, the investor may reduce the chances of purchasing the instruments for being too risk averse.

Table 5 presents the same comparative income analysis between the T-bill portfolio and the T-note plus inverse floater portfolio leveraged at a ratio of 1.5. Results and inferences similar to Table 4 can be drawn.

Conclusion

Each debt security has different risk characteristics. Similarly, each portfolio manager has investment objectives that reflect the type of risks that he is willing to bear. What is considered risky for one portfolio manager may not pose significant risks to another, depending on the purposes for which the fund is operated.

Portfolio managers should be able to choose those investments that best fit the desired risk profile in a given economic environment. If the manager attempts to eliminate all risks, including risks that do not impinge on the achievement of the investment objectives for the fund, he will be sacrificing returns without a commensurate reduction in the relevant risks to the fund or its participants.

For a portfolio manager who adopts a hold-to-maturity strategy and also has to achieve a consistent level of income, the inclusion of an inverse floater may have a positive contribution to the stabilization of the income of a portfolio of debt securities. And for the market timer, we have shown evidence that portfolio income can even be enhanced if initial spreads between 3-month and 5-year Treasuries are sufficiently high. However, historical performance is no guarantee of future performance, even if initial spreads are high.

Appendix 1¹⁷

One-factor stochastic interest rate models assume that a single factor captures all of the relevant information needed to value debt securities. This implies that all bond returns are perfectly correlated. In reality, the movement of returns on long-term bonds and short-term bonds are correlated but not perfectly. There are times that returns on long-term and short-term bonds may move in an opposite direction, causing a “twist” in the yield curve. Two-factor models are more capable to capture a wide range of possible movements along the yield curve. Two-factor models assume that the relevant information needed to value bonds of any maturity is contained in the two state variables.

The Schaefer and Schwartz Model

The Schaefer and Schwartz (“SS”) model involves two state variables. The consol rate is one of the state variables, and is defined as the yield on a bond that has a constant continuous coupon and infinite maturity. The second state variable is taken to be the spread between the instantaneously riskless rate and the consol rate. The short rate can be derived once the consol rate and spread are known, which are assumed to be *orthogonal*.¹⁸ Indeed, using the arbitrage arguments, the rates for bonds of any maturities can be calculated.

Calibration of Model Parameters

Both the consol rate and the instantaneously riskless rate are theoretical formulations and do not exist in practice. The Constant Maturity Treasury (CMT) series published by the Federal Reserve Bank are used as proxies. The 3-month CMT and 30-year CMT, respectively the shortest and longest maturity of the series, are used to approximate the instantaneous short-rate and consol rate. These are weekly data and are first converted into a continuously compounding basis before calculating the spread. The long-term mean spread is derived by averaging the 10-year historical spreads from the period from January 1982 through December 1991.

The speed of the mean reversion is estimated by regressing the change of the spread at time t on the gap of the spread at time $t-1$ from its long-run mean. The volatility

¹⁷ A more detailed appendix is available from David Babbel.

¹⁸ The SS model is an approximate analytical solution to a two-state variable model of the term structure similar to the one proposed by Brennan and Schwartz [1979, 1980, 1982]. Unlike the Brennan and Schwartz model, which was based on the consol rate and the short rate, the SS model is based on the consol rate and the spread which are assumed to be *orthogonal*. This assumption was first studied by Ayres and Barry [1979, 1980] and supported by empirical work of Schaefer [1980] and Nelson and Schaefer [1983].

of the long rate and spread are calculated using the 2-year weekly historical data for 1990 and 1991. Given a 5-year investment horizon in the simulation analysis, the market price of the “spread risk” is set to equate the 5-year CMT rate generated from the model to the 5-year CMT rate observed in the marketplace at the start of the simulation.

Simulation of Future Interest Rates

Using the 3-month and 30-year continuously compounded CMTs on December 31, 1991 — the start date of the 5-year simulation period, the initial spread is established. The future spread and the long rate are simulated every month forward for five years, from which the continuously compounded short rate is then calculated. The short rate is then converted back to a simple interest basis.

As mentioned earlier, once the consol rate and the spread are ascertained, the rates for any maturities can be calculated by using capital market equilibrium arguments. The process normally involves solving a second order partial differential equation. The analytical approximation approach developed by Schaefer and Schwartz is used to calculate the rates for other maturities once the consol rate and the spread are determined.

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	Investment Strategy	Mean	Standard Deviation	Deviation Relative to Zero Coupon Bond
Portfolio 1	T-Bill	32.57	8.23	8.48
Portfolio 2	T-Note	37.72	1.38	3.40
Portfolio 3	Inverse Floater	45.53	5.11	12.04
Portfolio 4	T-Note & Inverse	39.67	0.39	5.07
Portfolio 5	Leveraged 1.5	43.22	4.51	9.71
Portfolio 6	Leveraged 1.94	46.34	8.23	14.32
Portfolio 7	Zero Coupon Bond	34.62	0.00	0.00

Portfolio 7 Zero Coupon Bond 34.62 0.00

Table 1

	Investment Strategy	Minimum	Maximum	Mean	Standard Deviation
Portfolio 1	T-Bill	23.63	75.50	47.07	20.46
Portfolio 2	T-Note	33.27	100.09	55.15	5.35
Portfolio 3	Inverse Floater	23.06	134.87	61.33	9.90
Portfolio 4	T-Note & Inverse	35.33	108.79	56.69	2.30
Portfolio 5	Leverage 1.5	22.41	133.62	61.51	8.32
Portfolio 6	Leverage 1.94	10.93	155.46	65.74	16.57
Portfolio 7	Discount Bond	34.61	119.20	58.05	0.00

Note: Total number of observations is 231. The standard deviations are computed relative to the Discount Bond benchmark returns.

Table 2

Lower Bound	Upper Bound	Number of Observations	% of Total
4%	Higher	0	0%
3%	4%	6	3%
2%	3%	57	25%
1%	2%	80	35%
0%	1%	39	17%
-1%	0%	34	15%
-2%	-1%	11	5%
-3%	-2%	4	2%
	Total:	231	100%

Table 3

Critical Level	Total Number of Occurrences with Spread > Critical Level	Minimum	Maximum	Mean	Standard Deviation	Number of Occurrences (T-Bill outperforms Note-Inverse)	%
None	231	-28.88	49.66	9.63	16.17	61	26.4%
2.0%	63	-25.32	46.53	17.42	9.65	11	17.5%
2.3%	30	-3.60	46.53	23.44	8.79	2	6.7%
2.4%	26	-2.28	46.53	24.97	9.39	1	3.8%
2.5%	18	-2.28	46.53	25.89	9.96	1	5.6%
2.6%	13	12.48	46.53	29.18	8.28	0	0.0%

Table 4

Critical Level	Total Number of Occurrences with Spread > Critical Level	Minimum	Maximum	Mean	Standard Deviation	Number of Occurrences (T-Bill outperforms Leveraged 1.5)	%
None	231	-43.32	74.49	14.44	24.26	61	26.4%
2.0%	63	-37.97	69.80	26.13	14.48	11	17.5%
2.3%	30	-5.40	69.80	35.15	13.19	2	6.7%
2.4%	26	-3.42	69.80	37.46	14.08	1	3.8%
2.5%	18	-3.42	69.80	38.83	14.94	1	5.6%
2.6%	13	18.71	69.80	43.77	12.41	0	0.0%

Table 5

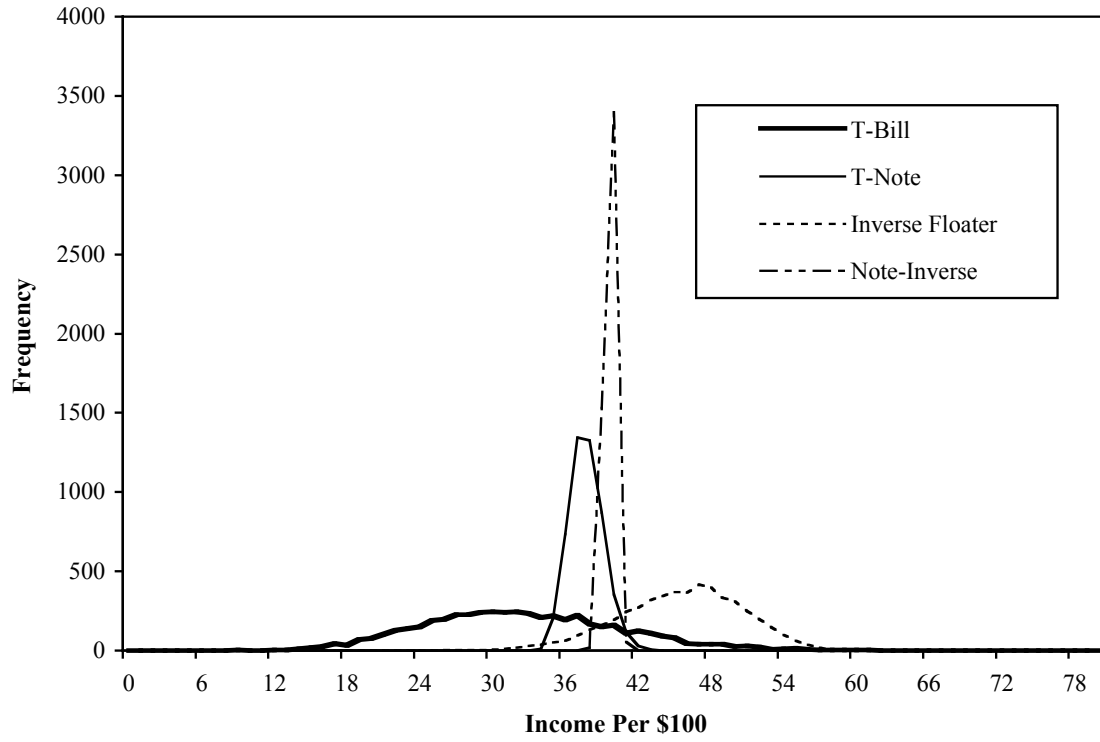


Figure 1

Income Analysis of Investment Strategies (Comparison Between Leveraged and Unleveraged Portfolios)

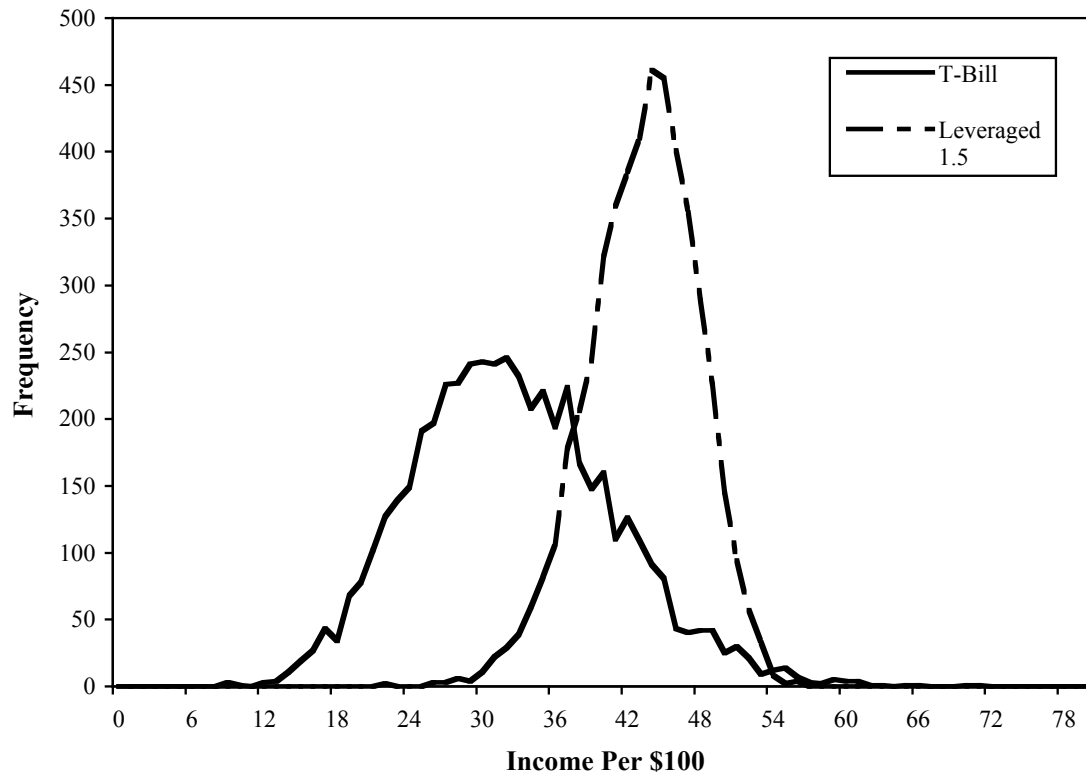


Figure 2

Income Analysis of Investment Strategies
(Comparison Between Leveraged and Unleveraged Portfolios)

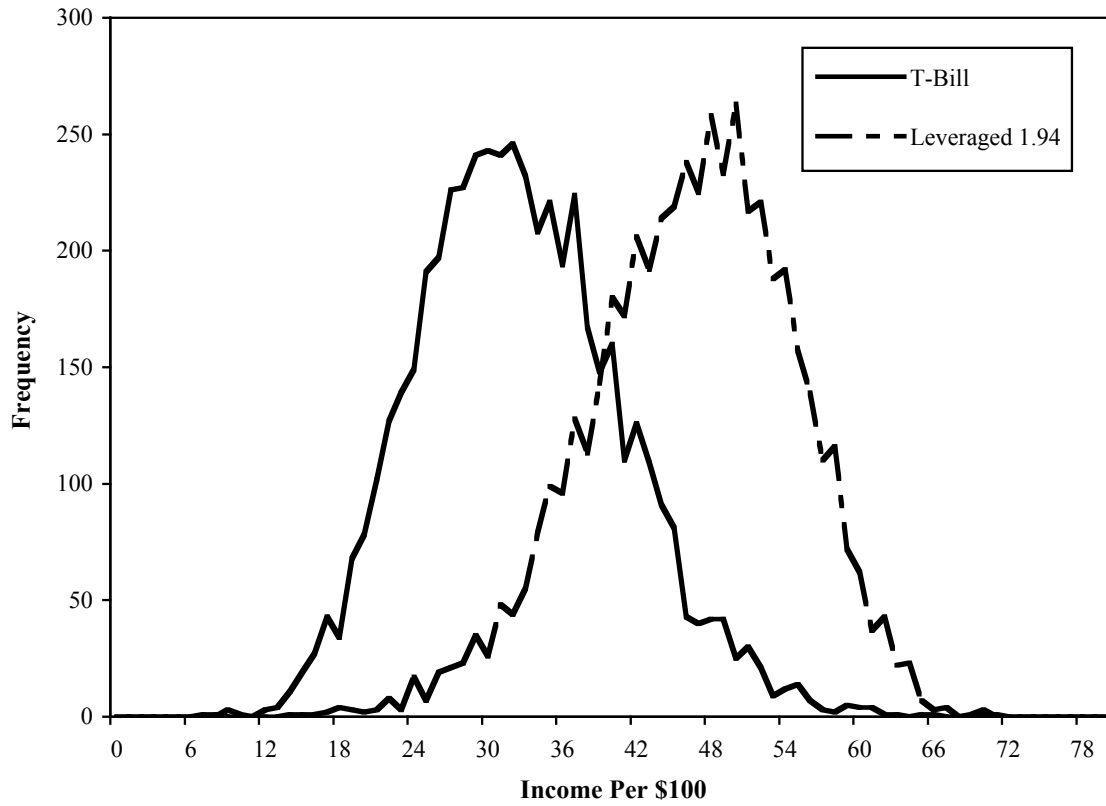


Figure 3

Stochastic Dominance Analysis of Investment Strategies (First Degree)

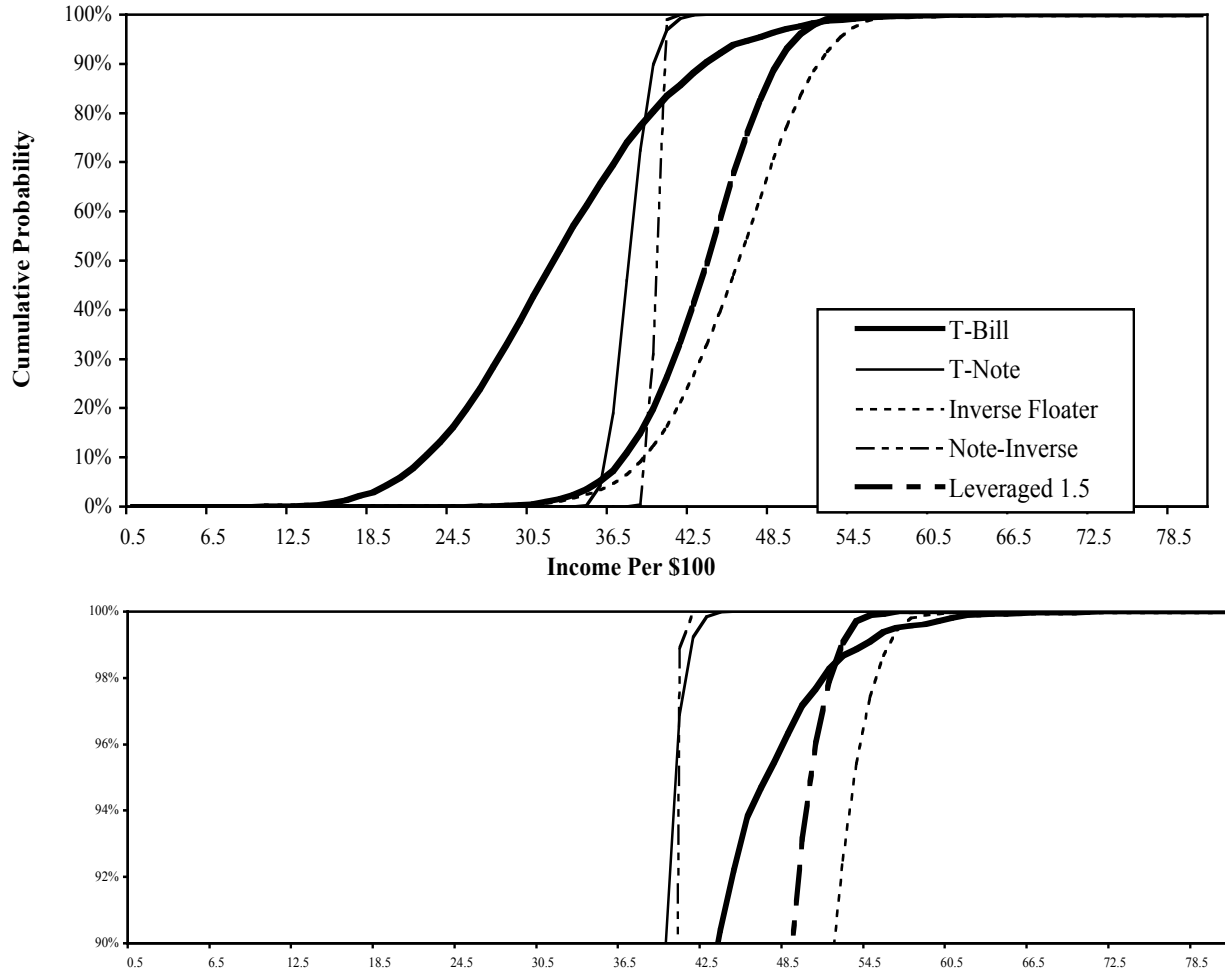


Figure 4

Stochastic Dominance Analysis of Investment Strategies (Second Degree)

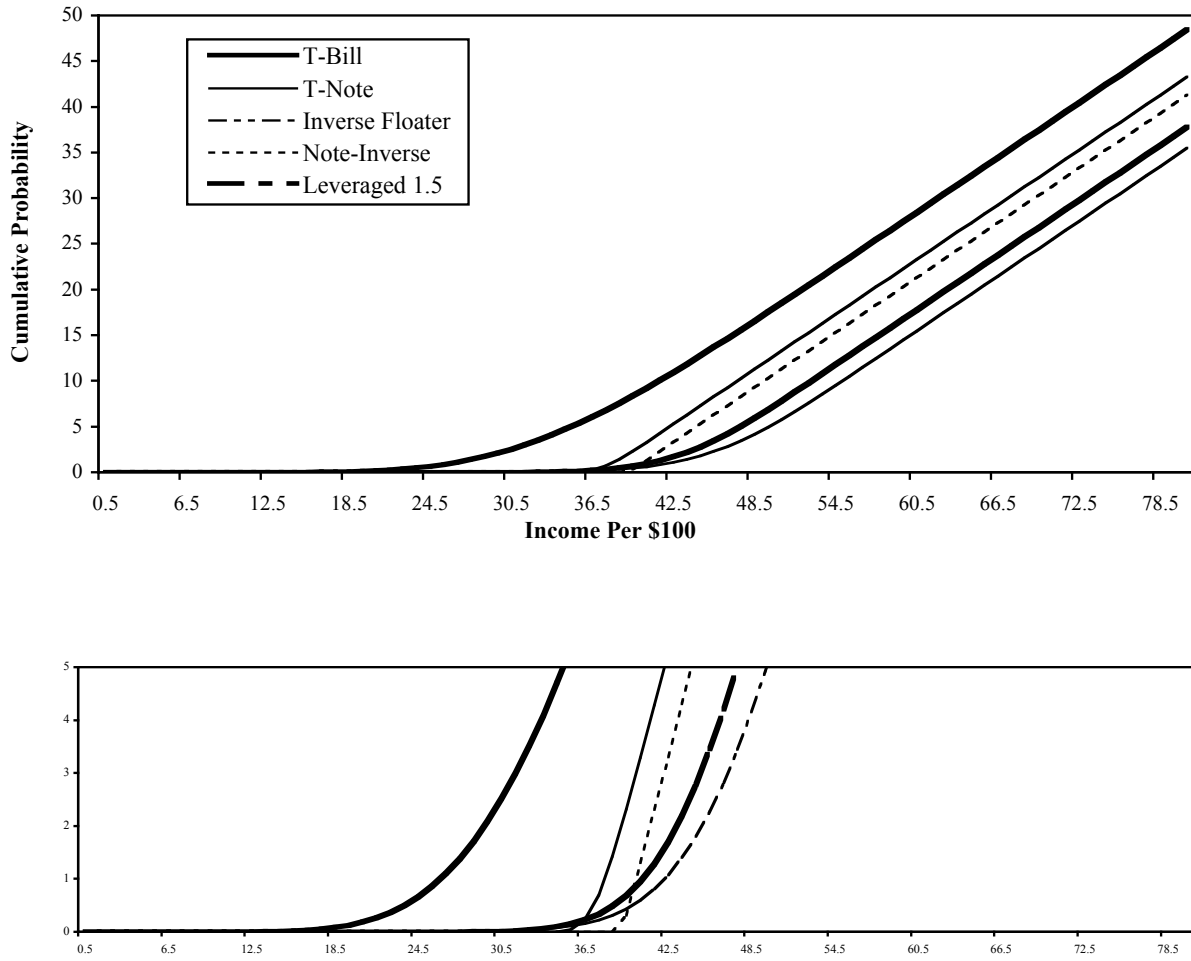


Figure 5

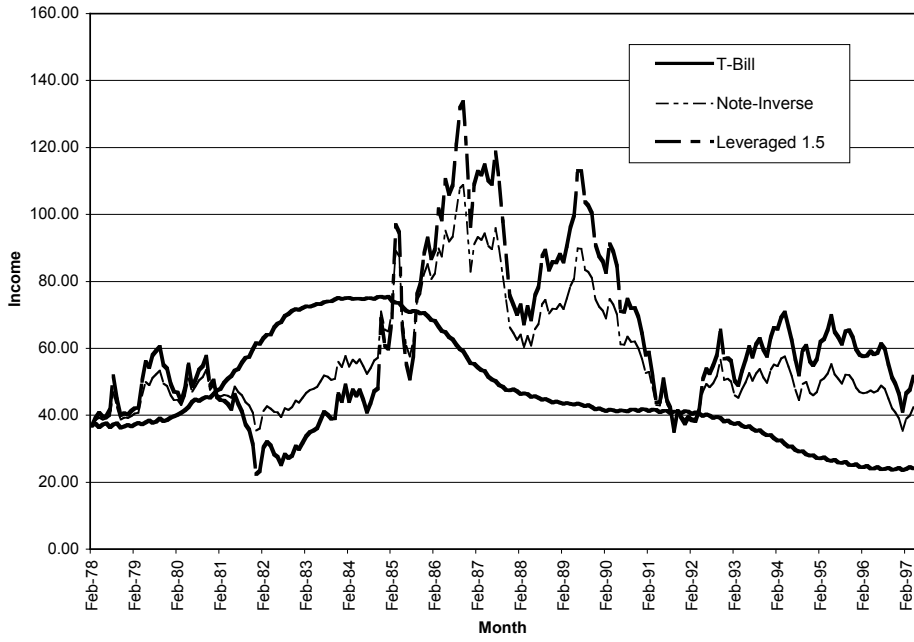


Figure 6

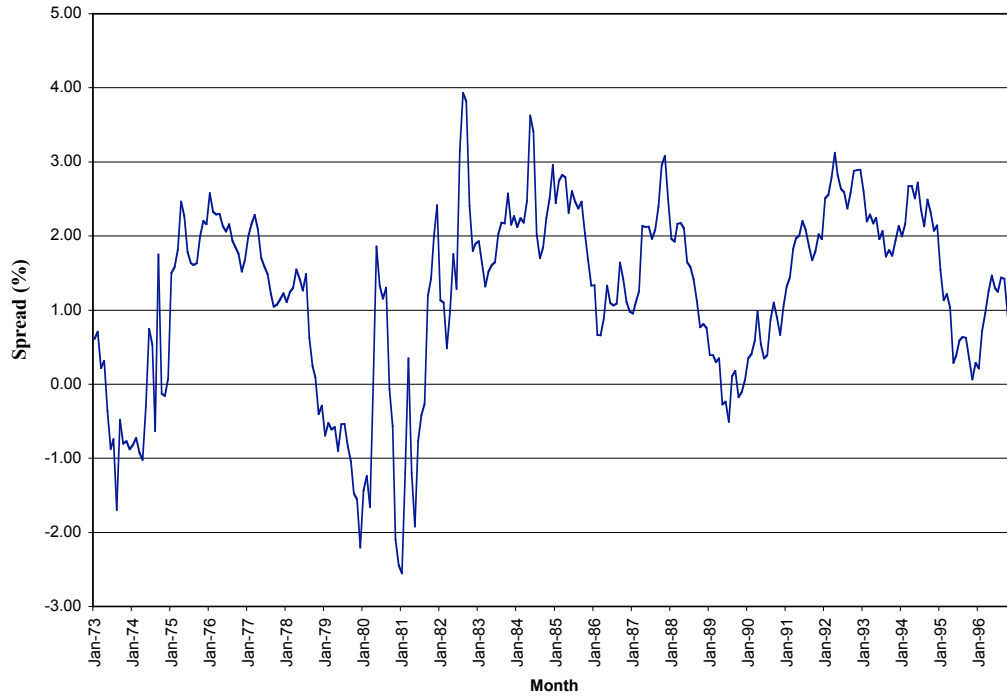


Figure 7