The Competitive Effects of Loyalty Discounts in a Model of Competition Implied by the Discount Attribution Test

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Abstract

This paper shows that the model of competition implied by the discount attribution test provides a simple, intuitive way to illustrate the procompetitive consequences of loyalty discounts through a comparison of the outcomes in the equilibrium in which sellers can only offer a single, unconditional price to the outcomes in an equilibrium in which they can offer loyalty discounts. If two sellers compete for sales to a buyer that has both contested and non-contested purchases, then, a smaller and less preferred seller has an advantage competing for sales of contested units because its more preferred and larger rival has higher lost profits on non-contested units. In an equilibrium in which sellers cannot offer loyalty discounts, this means that the more preferred seller does not compete for contested sales. In the loyalty-discount equilibrium, the larger seller has an advantage because the smaller seller has to offer a lower price to offset the larger lost discounts a buyer would suffer if it chose its loyalty discount offer rather than the larger sellers’ offer. This means that allowing sellers to offer loyalty discounts reduces the sales of the smaller, less preferred seller. However, buyers benefit when sellers can offer loyalty discounts because the ability to offer loyalty discounts induces both sellers to compete more aggressively and lowers buyers’ costs. While loyalty discounts can cause a smaller seller to lose sales, they do not foreclose the smaller seller if its larger rival’s discounts pass the discount attribution test.

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I. INTRODUCTION

There has been considerable debate among economists and legal scholars about the competitive consequences of loyalty discounts. The type of discounts that have raised the most concern are all-units discounts offered to intermediaries that give buyers a lower price for buying some threshold volume of purchases or share of purchases from a seller and a higher price for any purchases below that threshold. While is a general consensus that these loyalty discounts can have both anticompetitive and procompetitive effects, there is no such consensus on the appropriate framework for assessing when a defendant’s loyalty discounts should be considered to violate the antitrust laws.

One of the main points of the debate on this issue has been the role that a price-cost test, such as the discount attribution test, should play in assessing whether loyalty discounts are anticompetitive. Several authors have pointed out both theoretical and practical problems with the discount attribution test. Nonetheless, some form of the discount attribution test often plays a role in litigation involving loyalty discounts, and some courts that have ruled on the competitive consequences of loyalty discounts have considered some version of the discount attribution test in assessing whether defendants’ loyalty discounts are anticompetitive.

Despite its potential flaws, the discount attribution test framework presents a simple, intuitive way to illustrate a potential exclusionary effect of loyalty discounts. The discount attribution test assumes that two sellers compete for sales to a buyer that has both contested and non-contested purchases. If so, a seller may be able to exclude its rival by offering discounts on the non-contested units, for which it faces no competition from the other seller, that are conditional on the buyer choosing to buy its contested purchases from the seller rather than its rival. To do so, a seller’s loyalty discounts must be sufficiently large that its rival will not be able to compete for contested sales because the buyer would be unwilling to forego the discounts to buy from the rival. To conduct the discount attribution test, one first attributes all of a seller’s

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4 See, e.g., Cascade Health Solutions v. PeaceHealth, 515 F.3d at 909.
discounts on non-contested units to the profits it earns on contested units. If the seller’s profits on contested units with the attributed discounts exceed the costs of selling the contested units, then the seller offering loyalty discounts can foreclose an equally efficient rival from making contested sales to buyers.

This paper shows that the discount attribution test also provides a simple, intuitive way to illustrate the procompetitive consequences of loyalty discounts. The discount attribution test is implicitly based on a model of competition which can be used for comparing outcomes in an equilibrium in which sellers offer loyalty discounts to the outcomes in an equilibrium in which sellers can only offer a single, unconditional price. This comparison illustrates the effect of loyalty discounts on competition and consumers.

When sellers compete for sales to buyers that have both contested and non-contested purchases, competition is limited by the fact that competing for contested sales means lowering the profits sellers earn on non-contested sales. If the sellers are equally efficient, the less preferred seller (i.e. the seller with a smaller value of non-contested sales) has an advantage over the more preferred seller because its cost of competing for contested sales is lower due to the fact that its lost profits on non-contested sales is smaller. When sellers cannot offer loyalty discounts, this means the more preferred seller does not compete for contested sales because the costs in terms of its lost profits on non-contested sales necessary to win contested sales exceed the profits it could earn on contested sales. In an equilibrium in which sellers can offer loyalty discounts, the more preferred seller competes for contested sales because it only loses profits on non-contested sales if it makes contested sales.

To make contested sales in an equilibrium with loyalty discounts, a seller has to offer loyalty discounts that make the buyer at least as well off as it would be if it accepted its rivals’ loyalty discount offer. The cost to the buyer of accepting either seller’s offer is that it loses discounts on the other seller’s non-contested purchases. Since a buyer’s lost discounts are larger if it accepts the less preferred seller’s loyalty discount offer, this gives the more preferred seller an advantage over the less preferred seller in competing for contested sales in that it can offer a higher discounted price than its rival and still win the contested sales.

Thus, while the less preferred seller has an advantage competing for contested sales in a single-price equilibrium, the more preferred seller has an advantage in the loyalty-discount
equilibrium. Importantly, the advantage the more preferred seller has in the loyalty discount equilibrium does not foreclose an equally or more efficient rival. In a loyalty-discount equilibrium, buyers will choose the loyalty discount offer from the seller with the lower production costs. If the less preferred seller has lower production costs, then it will make contested sales with and without loyalty discounts. If the sellers are equally efficient, then they will split sales of contested units in a loyalty-discount equilibrium.

If the less preferred seller is equally or more efficient, then buyer costs are lower in a loyalty-discount equilibrium than in a single-price equilibrium. The equilibrium in which sellers can only offer a single, unconditional price is inefficient because the more preferred seller does not compete to make contested sales. Allowing sellers to offer loyalty discounts offsets this market failure by inducing the more preferred seller to compete for contested sales, and this transfers to buyers the profits the less preferred seller earns as a result of that inefficiency. Competition in the loyalty-discount equilibrium effectively induces sellers to price contested units at their marginal costs.

If, however, the more preferred seller’s production costs are sufficiently lower than its rival’s production costs, the more preferred seller will make contested sales in both the single-price equilibrium and the loyalty-discount equilibrium. In this case, less preferred seller offers more of a constraint on the price of the more preferred seller in the single price equilibrium where is has more of an advantage. Thus, while loyalty discounts do not cause the less preferred seller to lose any sales, buyer costs are higher with loyalty discounts.

These results are specific application of more general results regarding of the effect of price discrimination in a duopoly. More generally, price discrimination can, under certain condition, induce sellers to compete more aggressively leading to lower prices for all buyers.\(^5\) While others have noted that loyalty discounts can operate as a form of price discrimination that benefit buyers by inducing sellers to compete more aggressively,\(^6\) these effects do not appear to have been fully appreciated. Examining the competitive effects of loyalty discounts through the discount attribution test framework provides a simple way to illustrate these effects. It also sheds


light on several of the claims often made by plaintiffs and their experts in litigation involving loyalty discounts.

First, plaintiffs and their experts often claim that there are no cost based efficiency justifications for loyalty discounts, particularly if they are share based discounts. In the discount attribution test model, there are two costs of competing for contested sales: production costs and costs in terms of lost profits on non-contested sales. A sufficiently large reduction in either of these two components can induce the more preferred seller to compete for contested sales. Allowing sellers to offer loyalty discounts reduces the more preferred seller’s cost of competing for contested sales because it reduces its lost profits on non-contested sales. This reduction in the costs in terms of lost profits can induce the more preferred seller to compete for contested sales. In this sense, allowing sellers to offer loyalty discounts has the same efficiency enhancing effect as a reduction in the more preferred seller’s production costs.

Second, plaintiffs and their experts frequently claim that loyalty discounts can foreclose equally efficient rivals because they prevent rivals from profitably competing for a range of different shares of a buyer’s purchases. In the model of competition implied by the discount attribution test, buyer costs are minimized when the buyer chooses one seller for all of its contested units. This analysis shows that loyalty discounts do not prevent equally efficient competitors from competing for the share of buyer’s purchases that minimize a buyer’s costs. Thus, the fact that may be a range of shares for which a rival cannot compete does not mean that the rival is foreclosed by loyalty discounts.

Third, plaintiffs and their experts often argue that loyalty discounts should be viewed as penalties because they increase the buyers’ cost of buying from the smaller rival. Economists have similarly noted that loyalty discount can act like a tax on purchasing from the rival. In the discount attribution test model, such a tax increases rather than decreases competition. The single-price equilibrium is inefficient because the more preferred seller does not compete for contested sales. Taxing or penalizing buyers for purchasing contested units from the less preferred seller induces the more preferred seller to compete for contested sales. This replicates the outcomes with loyalty discounts, so buyers benefit from the tax.

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Fourth, plaintiffs and their experts often claim that competing by charging a single price represents competition on the merits, while competing through loyalty discounts is not competition on the merits. This analysis shows that this claim is incorrect. The less preferred seller has an advantage competing for contested sales in the single-price equilibrium that is unrelated to lower costs or greater demand. In fact, a less preferred seller can have higher sales than the more preferred seller in a single-price equilibrium even if it has higher costs and is less preferred. Loyalty discounts transfer to buyers the profits the less preferred seller earns as a result of that advantage.

This analysis does not rule out the possibility that sellers could use loyalty discounts to foreclose rivals and harm competition and consumers when there are economies of scale. However, the direct effect of loyalty discounts is to increase competition and benefit buyers, and any potential harm to competition or consumers would be indirect. Second, loyalty discounts that pass a discount attribution test cannot foreclose equally efficient rivals from competing for sales that minimize the buyer’s costs, so it is unlikely that such discount could reduce the sales of and equally efficient rival sufficiently to harm competition.

II. BACKGROUND ON ATTRIBUTION TEST FRAMEWORK

This section provides a brief description of how the discount attribution is applied in practice and an economic interpretation of the results of the test. It also surveys the critiques of economists and legal scholars of the discount attribution test.

A. Interpreting the Results of the Discount Attribution Test

The standard application of the discount attribution test calculates the profits a supplier earns on sales of contested units after subtracting the profits it lost by giving discounts on sales of non-contested units.

Consider the following example. Suppose there are two sellers, A and B, that compete for sales to buyer that has both contested and non-contested purchases. One might imagine that the two sellers are Intel and AMD and that the buyer is an OEM that sells computers to two types of end customers: some that strongly prefer Intel processors and some that are indifferent between computers with the two processors. Suppose a buyer purchases 80 units of a product sold by the two sellers. Assume seller A offers the buyer a price of 10 if the buyer purchases less than 80
units from seller A and 20 percent discount if it buys all 80 units from A. Suppose the buyer accepts that offer and buys 80 units from seller A and 0 units from its rival. Suppose also that if the buyer had not accepted seller A’s offer it would have bought only 40 units from seller A. In this example, the buyer (an OEM) has 40 non-contested units for seller A (Intel) and an additional 40 units that are contested between it and seller B (AMD). If seller A’s marginal cost per unit is a constant 5, then it earns a profit of 3 per unit on the contested sales for a total profit of 120. To win those contested sales, it offered discounts of 2 per non-contested unit or a total of 80.

In this example, the seller passes the discount attribution test because the 120 it earned in profit on contested sales exceeds the 80 in profit it gave up on non-contested sales. However, if the discount offered by seller A was 30 percent, then it only earns a per-unit profit of 2 on the contested units for a total of 80, but it gave up 3 per unit on non-contested unit for a total of 120. In this case, the seller would fail the discount attribution test.

More generally, one can write the formula for the discount attribution test as:

\[ Q^c(P^a_n(1-D) - C_a) - Q^a_nP^a_nD > 0 \]

Where \( Q^c \) is the number of contested units, \( Q^a_n \) is the number of contested units for seller A, \( P^a_n \) is the price it offers if the buyer does not commit to buying the contested units, \( D \) is the percentage discount the seller offers if the buyer does commit, and \( C_a \) is the seller’s marginal cost. If the term on the left hand side is positive, then seller A passes the discount attribution test for that buyer.

There are two possible interpretations of what the results of the discount attribution test tell us. First, one can interpret the results as telling us whether seller A has offered a below cost discount. In other words, it tells us whether the seller may be acting predatorily by offering a discount on non-contested units that is greater than the profits it earns on contested units. There is, therefore, some threshold discount percentage above which the seller’s profit from offering the discount is negative. Using, (1) that threshold discount is:

\[ D = \frac{Q^c(P^a_n - C_a)}{P^a_n(Q_c + Q^a_n)} \]

The threshold discount depends on both the per-unit margin on non-contested sales and on the share of contested units in contested and non-contested sales.
Second, the results of a discount attribution test can be interpreted as telling us whether an equally efficient rival could profitably compete for sales of the contested units. To see this, consider the example above. Suppose seller B also has a marginal cost of 5 per unit. For seller B to compete for contested sales, it has to offer a price that makes the buyer indifferent between buying from it and losing the discount on non-contested sales from seller A. The lowest price seller B could charge and still be profitable would be its marginal cost. If the buyer accepts seller A’s offer, it pays 8 for 80 units from A for a total of 640. If the buyer purchased its contested units from B at a price equal to B’s marginal cost, it pays 5 for 40 units from B and 10 for 40 units from A for a total of 600. Thus, seller B can profitably compete for contested units. However, if seller B’s marginal costs were 6, the buyer’s cost would be 640 if it rejected seller A’s loyalty discount offer. Thus, if seller B’s costs were 6 or higher, then it would not be able to profitably compete.

Since seller B could not profitably sell at a price below its marginal costs, one can interpret the discount attribution test as telling us whether B’s marginal cost is below the threshold level that would allow it to offer the buyer a price sufficient to compensate the buyer for forgoing A’s loyalty discounts. Using (1), that threshold cost level is given by:

\[
C = \frac{Q^c p_a^n (1-D) - Q^n p_a^n D}{Q^c}
\]

While this paper focuses on single product loyalty discounts, in a case involving bundled discounting, the discount attribution test is essentially the same. Suppose the seller offering loyalty discounts sells two products: one in which has a monopoly and another in which it competes with a rival. In this case, the non-contested sales are purchases by the buyer of the monopoly product, and the contested units are the purchases of the product that both sellers sell. The seller selling both products offers a bundled discount if the buyer buys both products from it. Application of the discount attribution test is same as in the single product case discussed above.

**B. Views on the Use of the Discount Attribution Test**

Economists and legal scholars have differing views on the role the discount attribution or similar test should play in assessing whether a seller’s loyalty discounts should be considered anticompetitive. These views are based on the two primary views of the appropriate framework for examining the competitive effects of loyalty discounts. Several authors have noted that the
debate among economists and legal scholars about the appropriate framework for analyzing the competitive effects of loyalty discounts has led to uneven treatment by courts and antitrust authorities.\(^8\)

One view is that the potential anticompetitive effect of loyalty discounts is similar to the competitive effect of exclusive contracts. According to this view, a market leading firms’ can use loyalty discounts to reduce the sales of smaller rivals by raising the cost of buying from a rival and foreclosing them from competing for some fraction of the market. If those smaller rivals cannot reach their minimally efficient scale or cannot cover their fixed/entry costs because of loyalty discounts, then those discounts can reduce the smaller firm’s ability to compete, or even prevent them from competing, if their sales are reduced sufficiently by its larger rivals’ loyalty discounts.\(^9\)

Since loyalty discounts can cause the smaller rival to lose sales even if the leading firm prices above its costs, then, according to this view, passing a discount attribution test does not tell us whether a seller’s loyalty discounts harm competition and consumers. Instead, any assessment of whether loyalty discounts are anticompetitive should be based on a rule of reason analysis similar to the assessment of whether exclusive contracts are anticompetitive. In other words, that assessment should be based on a comparison of the adverse effect of loyalty discounts on rivals’ ability to compete and of the procompetitive impact of the loyalty discounts. While proponents of this view have recognized that loyalty discounts can have procompetitive effects, they tend to emphasize effects such as enhancing promotional effort rather than the procompetitive effects identified in this paper.

A second view is that loyalty discounts are a natural outcome of the competitive process, and they generally benefit consumers even if they cause smaller rivals to lose sales.\(^10\) Thus, even though it is possible that a seller could use loyalty discounts to foreclose its rivals and acquire monopoly power, sellers will have an incentive to use loyalty discounts even when they have no intent to monopolize. Thus, according to this view, loyalty discounts, like discounting more


generally, should be considered anticompetitive only when there is evidence that a seller’s discounts were predatory. Proponents of this line of reasoning suggest that subjecting firms to antitrust scrutiny for engaging in a form of competition that almost always benefits consumers can chill firms’ incentives to engage in procompetitive activity.¹¹ Since, as discussed above, failing the discount attribution test can be an indication that sellers are pricing below costs on the contested units, failing the discount attribution test for a large share of buyers should be a necessary, though not sufficient, condition for a claim that a seller’s loyalty discounts anticompetitive.¹²

Proponents of both views have noted that implementing the discount attribution test can often be difficult in practice. One major difficulty is that the portion of an intermediary buyer’s purchases that are contested and non-contested units is not generally known. If one implements the discount attribution test based on an incorrect estimate of what portion of a buyer’s purchases are contested, then the results of the discount attribution test can be incorrect. Another issue is the implicit assumption that the only reason a seller would offer discounts on non-contested units is to win sales of contested units. Offering buyers lower prices will typically induce buyers to purchase more units. As a result, the discount attribution test will typically overestimate the profits lost on non-contested units, so loyalty discounts will generally be more profitable than calculated by a simple discount attribution test.

Proponents of the first view have argued that the practical difficulties support their belief that the discount attribution test is uninformative. Proponents of the second view argue that these problems indicate that passing the test should be a safe harbor and that failing the test does not necessarily imply a predatory intent.

¹² In addition, proponents of this view argue that loyalty discounts should only be considered anticompetitive if sellers likely can recoup the losses on those buyers that fail the test by charging monopoly prices in the future. U.S. DEP’T OF JUSTICE, COMPETITION AND MONOPOLY: SINGLE-FIRM CONDUCT UNDER SECTION 2 OF THE SHERMAN ACT 107 (2008), available at http://www.justice.gov/atr/public/reports/236681.htm
III. USING THE DISCOUNT ATTRIBUTION TEST MODEL TO ILLUSTRATE THE COMPETITIVE EFFECTS OF LOYALTY DISCOUNTS

This section uses the discount attribution test model to illustrate the effects of loyalty discounts on buyers and sellers. It first specifies a model of competition based on the discount attribution test framework, and then solves for the outcomes in a single-price equilibrium and a loyalty-discount equilibrium under the assumption that the two sellers are equally efficient. The results indicate that loyalty discounts benefit buyers. Allowing sellers to offer loyalty discounts induces the more preferred seller to compete for contested sales which forces the less preferred seller to offer lower prices. This section concludes by discussing the implication of the results for assessing claims often made by plaintiffs and their experts in loyalty discount cases.

A. Specifying Model of Competition Based on the Discount Attribution Test

The discount attribution test discussed above is based on a series of assumptions about buyers’ purchases and about the nature of competition between sellers. A model of competition based on the discount attribution test would have the following features.

First, the discount attribution test framework contemplates two sellers with constant marginal costs: 1) a market leader that is providing loyalty discounts; and 2) a smaller rival that may or may not be offering loyalty discounts itself. Call these two sellers A and B respectively, and let $C_a$ and $C_b$ be their marginal costs.

Second, there is a least one buyer that purchases contested units and some non-contested units from at least one seller. Let $Q^n_a$ and $Q^n_b$ be the number of non-contested units from each seller and $P^n_a$ and $P^n_b$ be the prices charged for those non-contested units. Assume that seller A is the more preferred seller in that all buyers’ expenditures on non-contested units are higher for A than B.\(^{13}\)

Third, a fraction of buyers’ purchases are contestable to both sellers, and let $Q^c$ be the number of contested units. The discount attribution test assumes that buyers would purchase the contested units from the seller offering the lowest price, so the implicit assumption is that buyers view the contested units from the two sellers as perfect substitutes for each other.

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\(^{13}\) More specifically, assume that $Q^n_a > Q^n_b$, and $Q^n_a P^n_a > Q^n_b P^n_b$. 

Fourth, the total quantity purchased is fixed, so demand is perfectly inelastic. In the discount attribution test framework, offering discounts on non-contested units or lowering prices on contested units does not increase the number of units sold. Thus, the only reason sellers offer discounts on non-contested sales is to win contested sales. This also means that there is no difference between discounts based on volume and discounts based on the share of units purchased.

Fifth, buyers and sellers know the number of non-contested and contested units for each buyer. To interpret failing the test as an indication that the seller has predatory intent, the seller has to know how many units are contested and non-contested. If the seller has imperfect information and believes the number of contested units is larger than the true number, then a seller may fail the test even if it believes it is offering profitable discounts.

Finally, the discount attribution test implicitly assumes that sellers negotiate discounts with buyers on an individual basis. This is related to the implicit assumption that sellers know the number of contested and non-contested units. If sellers could only offer a single loyalty discount across buyers with different contested and non-contested units, one could not interpret the failure of the test at any individual buyer as an indication that the seller had a predatory intent.

What type of preferences would generate the contested and non-contested demand implied by the discount attribution test? One can imagine buyers as intermediaries, such as retailers, that purchase for end customers that buy one unit of each seller’s product. The retailer has two types of customers: choosy customers who strongly prefer buying one seller’s products and non-choosy customers who buy based only on price. Non-choosy customers have an equal willingness to pay for both seller’s products, so units for these customers are contested. For the retailer to have non-contested demand for A’s product, then there must be a set of customers who value A’s product more than B’s and whose willingness to pay for B’s product is less than the price the retailer would charge if it paid B’s marginal costs for the products. If so, then even if seller B sold to the buyer at its marginal cost, these choosy customers would not be willing to buy B’s products. This means that purchases for these customers are non-contested in that the price seller A could charge for these units is not constrained by seller B.
For example, suppose there is a set of a retailer’s customers with a willingness to pay for a unit of A’s product of 11 and a willingness to pay for B’s product of 5. Assume also that the retailer charges customers a 10% mark-up. If seller B had a marginal cost of 5, those customers would not be willing to buy B’s product even if B charged the retailer its marginal costs. At a post mark-up price of 5.5, these customers would be unwilling to buy B’s products, so A could charge prices unconstrained by the willingness of these customers to substitute to B’s product. If seller A only wanted to sell units to the retailer for those customers, then it could charge the retailer a price of 10.

The assumption that number of units purchased by buyers is fixed means that the model would be better suited for analyzing the effect of loyalty discounts in markets where demand is more inelastic such as a hospital that buys medical products for different departments or different doctors. Alternatively, the two sellers could be suppliers of therapeutically equivalent drugs that compete for sales to pharmacy benefit managers for plans providing pharmaceutical benefits. Some choosy plan members may have a strong preference for one of the two drugs, while other non-choosy plan members may view them as substitutes for each other and buy whichever drug is on the lowest price formulary tier. Because demand is relatively inelastic, offering lower prices to plans is not likely going to have a large impact on the total number of units purchased.

C. The Impact of Loyalty Discounts When the Two Sellers Are Equally Efficient

To assess the impact of allowing sellers to offer loyalty discount, this section first describes the equilibrium outcomes in the discount attribution test model if equally efficient sellers can only offer a single, unconditional price. It then solves for the equilibrium outcomes if equally efficient sellers are allowed to offer loyalty discounts, and compares the outcomes in the two settings.

1. Single-price equilibrium

If each seller has non-contested units for a buyer, they are guaranteed a profit equal to the number of non-contested units times the per unit margin on sales of those units. If sellers can

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14 There have, in fact, been several loyalty discount cases involving sales of medical products to hospitals and drugs through formularies. See, for example, Eisai Inc. v. Sanofi-Aventis U.S., LLC, No. 08-4168 (MLC), 2014 U.S. Dist. LEXIS 46791, at *73-75- (D.N.J. Mar. 28, 2014).
only offer the buyer a single, unconditional price, then selling at any reduction in its price below the profit maximizing non-contested price will reduce its profits on non-contested sales. As a result, profits on non-contested units act to limit the willingness of sellers to compete for contested sales. The lowest price a seller will offer for the contested units will be one that gives it the same profit it would earn if it only sold the non-contested units.

Consider the following example. Suppose both sellers’ non-contested price is 10 and their marginal cost is a constant 5 per unit. Assume a buyer purchases 40 non-contested units for seller A, 40 contested units, and 20 non-contested units for seller B. A’s profit, if it only competes for non-contested units, is \(200 = 40 \times (10 - 5)\). This means that A would only offer a price to compete for contested sales that would give it a total profit of at least 200. The price that generates a profit of 200, if A sells both contested and non-contested units, is 7.5. (Table 1 shows the relevant values in the single-price equilibrium (“SPE”).)

Below, I refer to the price that generates the same profits as if the seller made only contested sales as the profit-neutral price. It is important to note that a seller’s profits can still be positive if it prices below the profit-neutral price. However, for prices below the profit-neutral price, profits are less than if sellers charged buyers a price that maximized profits on sales of non-contested units only.

The profit-neutral price for A depends on 2 factors: 1) its marginal production costs per unit of 5; and 2) the per-unit cost in terms of profit lost on non-contested units of 2.5. The latter factor itself depends on the product of two components: 1) the per unit margin on non-contested units of 5; and 2) the share of non-contested units in contested plus non-contested units of .5. More generally, defining \(P^*_i\) as the profit-neutral price for seller i, then:

\[
(2) \quad P^*_i = C + \frac{q^n_i(p^n_i - c)}{(q^n_i + q^c_i)}
\]

Using (2), the lowest price that seller B would offer is 6.67. Seller B’s profit-neutral price is lower because its lost profits on non-contested units is lower due to the fact that the share of non-contested units in contested plus non-contested units is only .33.

In the discount attribution test framework, buyers view the two sellers as perfect substitutes for their contested units. This means the buyer offering the lower price will win the contested sales, and the other seller will only make sales for the non-contested units.
Consider the pricing decisions of the two sellers. Seller A’s profits are lower if it charges below 7.5 than if it charges 10, so it will not offer a price below 7.5. Seller A also knows that if it prices at 7.5, it will be profitable for seller B to price just below 7.5. If so, seller B would win all the contested sales, and A’s profits would be lower than if it priced at 10. As a result, seller A will charge a price of 10, and it will not compete for contested sales. Seller B knows it can win all the contested sales if it prices just below 7.5, and it will be profitable to do so because 7.5 is greater than its profit-neutral price of 6.67.

Thus, in an equilibrium in which sellers offer a single, unconditional price, A charges 10, and B charges just below 7.5. The buyer pays 400 for the 40 units that it buys from A and 450 for 60 units it buys from B for a total of 850. Seller A earns a profit of 5 per unit on 40 units for a total of 200, seller B earns a profit of 2.5 per unit on 60 units for a total of 150 which is higher than the 100 it would have earned if it only sold contested units.

The appendix shows that the buyer costs in the single-price equilibrium can be written as:

\[ (3) \, \alpha Q_A^n P_A^n + \alpha Q^C C + Q_B^n P_B^n \]

where \( \alpha = P_A^* / P_B^* \). The ratio of the two profit-neutral prices depends on the differences in the number of non-contested units. As A’s non-contested units rises relative to B’s, its cost of competing for non-contested rises which raises the price that B can charge buyers.

\[
\begin{array}{|c|c|c|}
\hline
& \text{Seller A} & \text{Seller B} \\
\hline
\text{Non-contested units} & 40 & 20 \\
\hline
\text{Contested units} & 40 & 40 \\
\hline
\text{Non-contested price} & 10 & 10 \\
\hline
\text{Marginal Cost} & 5 & 5 \\
\hline
\text{Profit-neutral price} & 7.5 & 6.67 \\
\hline
\text{Price in SPE} & 10 & 7.5 \\
\hline
\text{Sales in SPE} & 40 & 60 \\
\hline
\end{array}
\]
| Profits in SPE | 200 | 150 |
| Buyer costs from seller | 400 | 450 |
| Buyer costs in LDE if it accepts seller’s loyalty discount offer | 800 | 800 |
| Price in LDE | 7.5 | 6.67 |
| Profits in LDE | 200 | 100 |
| Buyer costs from seller if it chooses A’ LD offer | 600 | 200 |
| Buyer costs from seller if it chooses B’ LD offer | 400 | 400 |

2. **Equilibrium with loyalty discounts**

Assume, instead, that buyers allow sellers to offer two prices: a lower price if the buyer commits to buying all the contested units from it, and a higher price if the buyer purchases contested units from its rival. One immediately obvious effect of allowing sellers to offer loyalty discounts is that it could induce seller A to compete for contested units. In the single-price equilibrium, seller A would not compete for contested units because if offer a price of 7.5 or higher it would lose profits on non-contested units without being able to make any contested sales. If A can offer a price of 7.5, conditional on it making the contested sales, and 10 if it does not, then it does not lose profits on non-contested sales if the buyer chooses B for its contested sales.

A second effect of allowing for loyalty discounts is that it changes the determination of which supplier wins the contested sales and the price necessary to do so. In the single-price equilibrium a seller simply needs to offer a price lower than its rival in order to win the contested sales. If sellers can offer loyalty discounts, then to win contested sales a seller must offer a discounted price that makes the buyer better off accepting its loyalty discount offer rather than its rival’s offer. This means that the seller with the lower profit-neutral price will not necessarily win the contested sales in the loyalty-discount equilibrium.
Suppose each seller offers its profit-neutral price conditional on the buyer giving it all the contested purchases and its non-contested price if it gives the contested purchases to its rival. In other words, seller A offers a price of 7.5 if it sells 80 units and a price of 10 if it sells less than 80, and suppose that seller B offers a price of 6.67 if it sells 60 units and a price of 10 if it sells less than 60. The cost to the buyer of buying 80 units from A and 20 from B is 7.5*80 + 10*20 = 800. The cost of buying 40 units from A and 60 units from B is 10*40 + 6.67*60 = 800. (See Table 1 for the relevant values in the loyalty-discount equilibrium (“LDE”)).

If both seller’s offer a loyalty discount at their profit neutral price, the buyer would be indifferent between choosing A or B for its contested sales. If either seller offered a price above its profit neutral price, then the other seller could offer a price just below it and capture all the contested sales. Thus, competition among the sellers will induce them to offer loyalty discounts equal to their profit-neutral prices. Since buyers would be indifferent between the two loyalty discount offers, the determination of which seller makes the contested sales is random. If there are a number of different buyers, then we would expect that the sales of contested units would be evenly split between the two sellers across all buyers. Seller A earns a profit of 200 independent of whether the buyer chooses its loyalty discount offer or B’s loyalty discount offer, and seller B earns a profit of 100 in either case.

The appendix shows, more generally, that each seller offers its profit-neutral price in the loyalty-discount equilibrium and that buyers are indifferent between purchasing their contested units from seller A or B even though seller B offers a lower price.

This means that the less preferred seller is worse off in the loyalty-discount equilibrium. In the single-price equilibrium, the less preferred seller makes the contested sales for all buyers. In the loyalty-discount equilibrium, both sellers make contested sales to some buyers. The less preferred seller also has to charge a lower price to compete for contested sales. Because it sells fewer contested units and charges lower prices when it does, seller B’s profit is lower in the loyalty-discount equilibrium than the single-price equilibrium.

In addition, buyers are better off in the loyalty-discount equilibrium. As shown in Table 1, the reduction in buyer costs is exactly equal to the reduction in seller B’s profit. In the single-price equilibrium, the more preferred seller does not compete for contested sales, so the less preferred seller can win contested sales by charging a price greater than its profit-neutral price. In
the loyalty-discount equilibrium, seller A does compete for contested sales, and this causes both sellers to offer their profit-neutral price. Effectively, loyalty discounts transfer to buyers the profits seller B earns because seller A does not compete in the single-price equilibrium.

D. Interpreting the results

The first difference between the loyalty-discount equilibrium and the single-price equilibrium is that seller A competes for contested sales in the loyalty-discount equilibrium when it would not do so in the single-price equilibrium. To see why, note, as I discussed above in reference to equation (2), the price a seller is willing to offer to compete for contested units depends on two factors: 1) the seller’s marginal production cost; and 2) the cost per unit to the seller in terms of lost profits on non-contested sales. If seller A does not compete for contested sales, it could be induced to compete for contested sales if either its production costs fall or the per-unit lost profit on non-contested sales falls.

The per unit lost profits on non-contested units depends on the number of contested units. As the number of contested units rises, the lost profit on non-contested units is spread out over a larger number of units which induces the seller to offer lower prices. In the single-price equilibrium, seller A knows that it cannot win the contested sales. As a result, it effectively assumes the number of contested units equals zero and simply offers its non-contested price.\(^\text{15}\) In the loyalty-discount equilibrium, seller A’s cost of competing for contested sales is lower because it only loses profits on contested units if it makes the contested sales, so the actual number of contested units affects the price it is willing to offer.

In addition to changing who competes for contested sales, allowing sellers to offer loyalty discounts also changes how they compete for contested sales. In the single-price equilibrium, the buyer chooses contested sales based only on the price offered by the two sellers. In the loyalty-discount equilibrium sellers compete by offering prices that make the buyer prefer its loyalty discount offer over its rival’s offer.

In the single price equilibrium, the less preferred seller has an advantage because its lost profits from competing for contested sales are lower. In fact, as shown in Table 1, the buyer purchases 60 percent of its units from seller B and 40 from seller A even though B is the less

\(^{15}\) Setting \(Q^c = 0\) in (2) yields \(P_a^* = P_a^n\).
preferred seller. In the loyalty-discount equilibrium, that advantage does not prevent seller A from competing for contested sales.

Moreover, in the loyalty-discount equilibrium, seller A has an advantage because the buyer has a higher cost of committing to seller B. For the same discounted price, buyers’ costs of committing to B are higher for two reasons. First, the buyer will prefer seller A over seller B for the same discounted price because the discounted price applies to both contested and non-contested units, and seller A has more non-contested units. Second, the premium the buyer has to pay for non-contested units is higher if it selects seller B’s loyalty discount offer rather than seller A’s offer, and this allows seller A to offer a higher discounted price.

For example, if both sellers offer a loyalty discounted price of 7.5 and a price of 10 if the buyer chooses its rival’s offer, then the buyer would prefer A’s offer because it would pay 7.5 for 80 units rather than pay 7.5 for 60 units if it accepts B’s offer. In addition, if it chooses B’s offer, it has to pay an undiscounted price of 10 for 40 units rather than pay an undiscounted price of 10 for 20 units if it accepts A’s offer. These two effects are captured by the two terms on the right hand side of (4) which illustrates the more general expression for the price seller A needs to charge to make the buyer indifferent between its offer and seller B’s offer if B offers its profit-neutral price.

\[
(4) \quad \widehat{P_A} = \frac{p^C_B (Q^C_A + Q^C_C)}{(Q^C_A + Q^C_C)} + \frac{p^n_A Q^n_A - p^n_B Q^n_B}{(Q^n_A + Q^n_C)}
\]

However, it is important to note that, even though loyalty discounts force seller B to lower its price, it is not foreclosed as a result of loyalty discounts. While seller B loses sales because of loyalty discounts, it is still able to offer a loyalty discount that makes buyers indifferent between its offer and seller A’s offer. In other words, loyalty discounts, offered at sellers’ profit-neutral prices, do not foreclose an equally efficient competitor from competing for contested sales.

Another way to think about why buyers are better off in the loyalty-discount equilibrium is that it effectively allows seller A to price discriminate. The inefficiency in the single-price equilibrium is due to the fact that seller A does not compete for contested sales. Seller A could be induced to compete for contested sales if it could charge lower prices for contested sales without having to reduce its profits on non-contested sales.
To see why allowing sellers to offer loyalty discounts effectively induces sellers to price discriminate, using (2), one can solve for the cost to the buyer in the loyalty-discount equilibrium as:

\[
(5) \quad Q_A^n P_A^n + Q_C C + Q_B^n P_B^n
\]

In the loyalty-discount equilibrium, buyers effectively pay the non-contested prices for the non-contested units but pay marginal costs for contested units. In other words, the ability to use loyalty discounts leads to prices that are equivalent to a situation in which sellers can price discriminate between contested and non-contested sales, and doing so induces sellers to compete prices for contested units down to their marginal costs.

Comparing (5) with (3) indicates that buyer costs are higher in the single price equilibrium because \( \alpha > 1 \). One can interpret \( \alpha \) as the premium buyers have to pay because seller A does not compete for contested sales in the single-price equilibrium. The comparison also shows that loyalty discounts benefit buyers because one seller is more preferred. If the number of non-contested units for each buyer were equal, then both sellers would have the same profit-neutral price because neither seller would have an advantage. In addition, if \( \alpha = 1 \), the buyer costs are same with and without loyalty discounts.

In general, allowing sellers to price discriminate will often cause inelastic demanders to pay higher prices and force more elastic demanders to pay lower prices. As a result, the net effect of price discrimination on buyer welfare is generally ambiguous. However, when sellers have different choosy buyers, the ability to price discriminate can often induce sellers to compete more aggressively for customers for whom they would not compete if they could not price discriminate, and this can lead to lower prices, or at least not higher prices, for all buyers. When the ability to price discriminate has this effect, economists have referred to this price discrimination leading to all-out-competition.\(^{16}\) Thus, allowing sellers to offer loyalty discounts benefits buyers in the discount attribution test framework because it leads to all-out-competition.

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E. Clearing up misconceptions about loyalty discounts

Plaintiffs’ and their economic experts in loyalty discount cases often make claims that are based on misconceptions regarding the competitive effects of loyalty discounts. The model described above helps shed light on some of those misconceptions.

1. Loyalty discounts have procompetitive justifications

Plaintiffs and their experts often argue that there are no cost based efficiency justifications for loyalty discounts, particularly market share discounts. The analysis above indicates that because loyalty discounts increase competition and induce both sellers to offer lower prices they have a clear procompetitive justification that is not cost based. Thus, the existence of a cost based efficiency justification is not necessary to establish that loyalty discounts have a procompetitive effect.

Nonetheless, the analysis above indicates that loyalty discounts have the same effect as an efficiency enhancement that reduced seller A’s production costs. According to (2), a seller’s profit neutral price depends on both its per unit production costs and the cost in terms of lost profits on non-contested units. An efficiency enhancement that reduces seller A’s production costs can induce seller A to compete for contested sales. Since the ability to offer loyalty discounts reduces the second term on the right hand side of (2), the ability to offer loyalty discounts has the same effect as an efficiency enhancement that reduces production costs.

2. Loyalty discounts do not foreclose rivals even if the discounts create a range of shares for which rivals cannot compete

Plaintiffs and their experts often claim that loyalty discounts foreclose rivals from competing if there are specific shares, or a range of shares, of a buyer’s purchases for which a rival cannot profitably compete. For example, if seller A offered a loyalty discount price of 7.5, seller B would not be able to profitably compete to sell the buyer 21 units. The buyer’s costs of purchasing 21 units from B at its marginal costs of 5 and 79 units at A’s non-contested price of 10 would equal 895, and this is more than the 800 the buyer would pay if purchased 80 units from A and 20 units from B. 17

This analysis shows that the existence of a range of shares for which seller B cannot compete does not mean that seller B is foreclosed. A buyer would never purchase between 21

17 In fact, seller B would not be able to compete for between 21 and 39 units.
and 59 units from seller B even in a single-price equilibrium because the buyer’s costs are lower if it gives all of its contested sales to one seller. Thus, without loyalty discounts, the buyer would only purchase either 20 or 60 units from seller B. Seller B could only be foreclosed from competing by loyalty discounts if it could not profitably compete for 60 units: the level of sales to the buyer’s that minimize the buyer costs.

3. **Increasing the buyer’s cost of purchasing from the less preferred seller increases competition**

   Plaintiffs and their experts often argue that loyalty discounts acts like a tax or penalty on buying from seller B and that this tax harms competition because it raises the buyer’s cost of buying from rivals. The analysis above indicates that raising the cost of buying from seller B increases rather than decreases competition.

   If seller A were prohibited from offering loyalty discounts, one way to induce seller A to compete for contested units would be to tax buyers for purchasing contested units from B. Taxing buyer purchases of B can induce seller A to compete for contested sales and force seller B to lower prices to compete. Thus, a tax can replicate the loyalty-discount equilibrium and benefit buyers. To see this, using the example in Table 1, suppose the buyer face a per unit tax, if it purchases more than 20 units from B, that is just above the difference between A’s profit-neutral price of 7.5 and B’s profit-neutral price of 6.67. If so, seller A would be willing to offer a single, unconditional price of 7.5, and it would sell 80 units. Seller B, knowing it could not compete for contested sales, would charge 10 and sell 20 units. This means that the tax would exactly replicate the loyalty-discount equilibrium and buyers would benefit. Stated differently, the tax or penalty associated with loyalty discounts induces sellers to compete more aggressively, so it increases rather than decreases competition and benefits buyers.

   Another way to think about this is that the advantage B has in the single-price equilibrium is due to a market failure arising from the fact that seller A can offer only a single, unconditional price. This inefficiency reduces the incentive for A to compete for contested sales, and allows seller B to charge a higher price. The less preferred sellers’ costs of competing for contested sales are higher in the loyalty-discount equilibrium but they were artificially low in the single-price equilibrium when A does not compete for contested sales. Thus, penalizing buyers
from buying from selling B offsets the inefficiency and induces seller A to compete for contested sales.

4. **Competition with loyalty discounts represents competition on the merits**

The analysis above sheds light on a claim plaintiffs and their experts often make that competition in a single-price equilibrium represents competition on the merits while competition with loyalty discounts does not. This claim is correct because competition in the loyalty-discount equilibrium is no less competition on the merits than in the single-price equilibrium. In the single-price equilibrium, seller B’s artificial advantage over seller A is not related to lower costs or greater demand. Instead, it is due to the fact that is has lower lost profits on non-contested units. Loyalty discounts eliminate this advantage and transfer to buyers the profits the less preferred seller earns as a result of that advantage.

5. **A single-price equilibrium would not exist without a prohibition on loyalty discounts**

Plaintiffs and their experts often claim that a dominant firm’s refusal to offer a single unconditional price is evidence of its intent to use loyalty discounts to acquire monopoly power. However, the analysis above indicates that, absent a prohibition on offering loyalty discounts, there would not be an equilibrium in which both sellers offer a single, unconditional price. Since the buyer and the more preferred seller can both make themselves better off by negotiating loyalty discounts, then we would not expect the more preferred seller to offer a single unconditional price. We would also expect the buyer to require that sellers make loyalty discount offers. A buyer and a more preferred seller have an incentive to negotiate loyalty discounts independent of whether doing so leads to monopoly power, so the failure to offer loyalty discounts is not evidence of an intent to acquire monopoly power.

IV. **THE IMPACT OF LOYALTY DISCOUNTS WHEN ONE SELLER IS MORE EFFICIENT**

The analysis above identified two effects of allowing sellers to offer loyalty discounts. First, loyalty discounts can induce more preferred sellers to compete to win contested sales. Second, the ability to offer loyalty discounts affects the conditions under which sellers can win contested sales. If sellers are equally efficient, this means that buyers are unambiguously better off in a loyalty-discount equilibrium.
This section asks whether the above results hold if sellers have different marginal production costs. The answer depends on whether the same seller makes the contested sales in the loyalty-discount equilibrium and in the single-price equilibrium, so I consider these scenarios in turn.

A. Seller B makes contested sales in both the single-price equilibrium and loyalty discount equilibrium

Suppose seller B’s marginal cost is 4 rather than 5, so it is the lower production cost seller. Seller A’s profit-neutral price is still 7.5. Even though seller B’s profit-neutral price falls to 6, the reduction in seller B’s costs has no impact on the single price equilibrium. Seller B still charges seller A’s profit-neutral price 7.5, and seller A still charges 10. Buyer costs still equal 850. Seller B’s profits increases to 210.

Suppose both sellers offer loyalty discounts equal to their profit-neutral price. Seller B offers the buyer a price of 6 if the buyer purchases 50 units from it and a price of 10 if it buys less than 50 units. If the buyer accepted this offer, its costs would equal 760 rather than 800. Buyer costs if A offers a loyalty discount at its profit-neutral price are still 800. This means that seller B makes contested sales in the loyalty-discount equilibrium. Thus, seller B makes contested sales with and without loyalty discounts.

To win contested sales in the loyalty-discount equilibrium, seller B can offer a price greater than its profit-neutral price of 6. It only needs to offer a price that sets the buyer’s cost equal to 800, its costs if it chose seller A’s loyalty discount offer at its profit neutral price. Using (4), that price equals 6.67 which is lower than price seller B charges in the loyalty discount equilibrium. Since buyers pay 10 for A’s non-contested units with and without loyalty discounts, the buyer’s costs equal 800 rather than the 850 it pays in the single-price equilibrium. In addition, seller B’s profits fall to 160, so allowing sellers to offer loyalty discounts transfers 50 of seller B’s profits to buyers.

The appendix shows more generally that when sellers can offer loyalty discounts, the buyer will choose the loyalty discount offer from the seller with lower production costs if both seller’s offer their profit-neutral price. In the single-price equilibrium, the seller with the lowest cost in terms of both production and the cost in terms of lost profits on non-contested sales sells the contested units. With loyalty discounts, the seller with the lower production cost necessarily
wins the contested sales independent of the level of lost profits on non-contested sales. Thus, even though seller A has an advantage over seller B in the loyalty-discount equilibrium, that advantage is not enough to offset seller B’s lower costs and prevent it from making contested sales.

Moreover, when the less preferred seller has lower production costs, buyer costs are lower in the loyalty discount equilibrium. Intuitively, seller B’s price is constrained more by seller A in the loyalty-discount equilibrium because of seller A’s advantage over B with loyalty discounts. Using (4), buyer costs in the loyalty-discount equilibrium are:

\[ (6) \quad Q_a^n p_a^n + Q_c^c C_a + Q_b^n p_b^n \]

Seller B effectively sells contested units at seller A’s marginal costs which are higher than its own. The constraint on the price seller B is willing to offer for contested units is its rivals marginal cost which are higher than its own. This also means that even if seller A has higher production costs and does not make contested sales with or without loyalty-discounts, it still constrains the prices charged by seller B.

These results also indicate that, when the less preferred seller has lower production costs, the more preferred seller can only make contested sales if it prices below its profit-neutral price. In other words, if seller A is the high production cost seller, it could only exclude seller B from making contested sales if it failed the discount attribution test.

**B. Seller B makes contested sales in the single-price equilibrium, and seller A makes them in the loyalty-discount equilibrium.**

Assume now that seller A’s costs per unit are equal to 4 and that seller B’s costs are 5, so seller B is the high cost seller. If so, then seller A’s per unit profit on non-contested sales is 6 rather than 5, so its total profit on non-contested sales is 240. The lowest price that A would be willing to offer to compete for contested sales would yield a profit of 240. Using (2), that price is 7 rather than 7.5.

While a reduction in seller A’s production costs reduces the price it is willing to offer to compete for contested sales to 7, that price is still higher than the 6.67 that seller B is willing to offer. As a result, seller A still will not compete for contested sales, so seller B makes the contested sales in the single-price equilibrium. The reduction in A’s costs does force B to charge
just below 7 rather than 7.5. As a result, it reduces the buyer’s costs to 820 and reduces seller B’s profit to 120. However, seller B makes contested sales even though it is less preferred and has higher production costs.

If the two sellers can offer loyalty discounts and both offer their profit-neutral price, then seller A will offer a price of 7 if it sells 80 units and a price 10 if it sells less than 80, and seller B will offer a price of 6.67 if it sells 60 units and a price of 10 if sells less than 60. The cost to the buyer of buying 80 units from A and 20 from B is 760. The cost of buying 40 units from A and 60 units from B is 800. Thus, if each buyer offered the profit-neutral price, the buyer would choose seller A for the contested purchases.

Seller A would not have to offer a price of 7 to make the contested sales. Instead, it just needs to offer a price low enough to make the buyer better off by accepting its loyalty discount offer rather than seller B’s loyalty discount offer of 6.67. This means that seller A must offer a price which sets the buyers cost just below 800. Using (4), if A offers a price of 7.5 if the buyer purchases 80 units, then the buyers cost is 800 because it buys 80 units from A at a price of 7.5 and 20 units from B at a price of 10. It will be profitable for seller A to offer a price just below 7.5, conditional on the buyer purchasing 80 units, so the buyer will purchase contested units from seller A.

In summary, seller A charges 7.5 for 80 units and earns a profit of 280 in the loyalty-discount equilibrium. This is greater than the 200 profit it earns in the single-price equilibrium. Seller B charges 10 for 20 units and earns a profit of 100 which is less than the 120 it earns in the single-price equilibrium. As in the previous case, loyalty discounts reduce prices, sales, and profits for the less preferred seller. Again, buyers are better off in the loyalty-discount equilibrium because their total cost falls from 820 to 800, and, as before, the reduction in buyer costs equals the reduction in B’s profits. The appendix shows that buyers are always better off in a loyalty-discount equilibrium than an single-price equilibrium if the more preferred seller has lower production costs but does not compete for contested sales in the single-price equilibrium.

Finally, seller A would pass a discount attribution test. Seller A’s profits are higher with loyalty discounts, so the discounts it offers must be profitable. In addition, even though seller B does not make sales of contested units in the LDE, seller B could price low enough to win the contested sales. If seller B offered any price below 6.67, it could make the contested sales, and it
would be profitable to do so because that price exceeds its marginal cost of 5. Thus, loyalty
discounts do not prevent seller B from profitably competing for contested sales. Instead, seller B
does not make contested sales because it is unwilling to give up the profits it earns on non-
contested sales. This is analogous to the situation in the single-price equilibrium in which seller
A does not make contested sales because it does not want to forego lost profits on its non-
contested units. In that setting, Seller B’s single price does not foreclose Seller A from
competing. Instead, it does not compete for contested sales because it unwilling to forego lost
profits on contested units.

In fact, seller B would have no incentive to offer a loyalty discount even if seller A does. Seller B knows that the price needed to win contested sales would lead to lower profits than if it only sold non-contested units, so it has no incentive to offer a loyalty discount. This is consistent with the fact that plaintiffs in loyalty discount cases often do not offer their own loyalty discounts.

C. Seller A makes contested sales in the single-price equilibrium and in the loyalty-
discount price equilibrium.

Assume, now, that seller A’s cost per unit is equal to 3 while seller B’s cost is still 5. If
so, then seller A’s profit-neutral price falls to 6.5. Since this is below seller B’s profit-neutral
price of 6.67, seller A can makes contested sales in the single-price equilibrium by charging a
price just below 6.67. In other words, even though seller A is the more preferred seller, its
production costs are sufficiently lower than B’s that A’s profit-neutral price is lower than B’s
profit-neutral price.

In the single-price equilibrium, the buyer pays 6.67 for 80 units from A and 10 for 20
units from B for a total of 733.33. This represents a significant reduction in costs compared to
the 820 shown in Table 3 when A’s production cost is 4. This large reduction in buyer costs
occurs because a small reduction in production costs induces seller A to compete for contested
sales.

As discussed above, changes in seller A’s production costs do not affect outcomes in the
loyalty-discount equilibrium because Seller B’s costs are the constraint on A’s prices. Seller A’s
profit maximizing price in the loyalty-discount equilibrium is still 7.5, and total buyer costs are
still 800. This means that buyer costs are higher in the loyalty-discount equilibrium than in the
single-price equilibrium. The appendix shows more generally, that buyer costs are higher in the loyalty discount equilibrium if the more preferred seller has lower costs and makes contested sales in both the single-price equilibrium and the loyalty-discount equilibrium.

To understand why buyers are worse off in the loyalty-discount equilibrium if seller A makes contested sales with and without loyalty discounts, note that buyers pay B’s non-contested price for B’s non-contested sales in both settings. As a result, the effect of loyalty discounts on buyers depends on whether A charges more or less for its non-contested units and contested units in the loyalty-discount equilibrium. We know that A charges B’s profit-neutral price in the single price equilibrium. As noted above, A can charge more than B’s profit-neutral price and still win contest sales in the loyalty-discount equilibrium because it is the more preferred seller. Intuitively, buyer cost are higher with loyalty discounts because seller B constrains seller A’s prices more in the single-price equilibrium where its advantage over seller A is greater.

Even though buyers can be harmed by the more preferred seller’s use of loyalty discounts if the more preferred seller makes contested sales in the single-price equilibrium and the loyalty-discount equilibrium, the harm to consumers does not flow from harm to rivals because seller B does not lose sales as a result of loyalty discounts. Instead, the harm to consumers is due to the fact that loyalty discounts are a form of price discrimination. As discussed above, price discrimination can have ambiguous effects on buyers’ welfare. In this particular setting, the ability to price discriminate does not increase competition sufficiently to benefit buyers.

V. SHOULD THE DISCOUNT ATTRIBUTION TEST BE USED TO ASSESS WHETHER LOYALTY DISCOUNTS ARE ANTICOMPETITIVE?

The analysis above shows how examining the impact of loyalty discount in the discount attribution framework provides a simple way to illustrate why loyalty discounts have a procompetitive effect. What are the implications of this analysis for assessing whether the discount attribution test itself should be used for assessing whether loyalty discounts harm competition and harm consumers?

To assess this question, it is useful to summarize the two main results. Loyalty discounts can increase competition because they induce the more preferred seller to compete for contested sales. Buyers can benefit from this increased competition because loyalty discounts transfer to
buyer the profits a less preferred seller earns in a single price equilibrium as a result of the fact the more preferred seller does not compete in a single price equilibrium.

A second important result is that loyalty discounts cannot foreclose an equally efficient seller. A less preferred seller that is equally efficient may have lower sales and profits in an equilibrium with loyalty discounts, but it is not foreclosed from competing for sales. Loyalty discounts can, however, foreclose a less preferred seller that is equally, or even more efficient, efficient if the more preferred seller’s discount is so large that it would fail a discount attribution test.

These results suggest that a discount attribution test can play two roles in helping to assess whether a seller’s loyalty discounts harm competition and harm consumers. These two roles correspond to the two views on the appropriate framework for assessing whether loyalty discounts are anticompetitive. The first version of the discount attribution test discussed above can be used to rule out whether loyalty discounts are being used to exclude an equally efficient seller. If we observe a more preferred seller offering discounted prices below its profit-neutral price, the profits the seller earns on contested sales are less than the profits it has given up on non-contested sales, so the seller may be using loyalty discounts to exclude its rivals.

The same two caveats discussed above regarding the extent to which the discount attribution test is a reliable indicator of whether a defendant’s loyalty discounts are predatory still apply. If the number of units purchased is not constant, then the discount attribution test framework will underestimate the less preferred seller’s sales and overestimate the more preferred seller’s sales in the but-for world. In addition, if a discount attribution test is based on an inaccurate estimate of the true number of contested units relative to non-contested units, than it may not provide a reliable indication of the profit-neutral price.18

In addition, one of the steps in assessing whether loyalty discounts are anticompetitive using the framework of exclusive dealing is the calculation of the foreclosure share. A buyer getting a loyalty discount is not necessarily foreclosed to a rival seller as a result of that loyalty discount, so the share of buyers getting loyalty discounts is not the appropriate measure of the

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18 Even if the share of a buyer’s purchases that were contested and non-contested were not known, one could still conduct a discount attribution test under different assumptions about the shares of contested sales to determine a range of possible shares under which an equally efficient seller would be excluded. One could then assess whether the evidence is consistent with the contested share falling in that range.
foreclosure share. Instead, the foreclosure share can be calculated by applying the second version of the discount attribution test discussed above on a customer-by-customer basis.

This analysis does not rule out the possibility that profitable loyalty discounts could harm competition and/or consumers. A more preferred seller’s use of loyalty discounts can contested sales of an equally efficient rival and would prevent a higher cost rival from making contested sales. If the reduction in sales causes the less preferred seller’s marginal costs to increase because it is below its minimally efficient scale or prevents it from being able to cover its fixed costs, then loyalty discounts can harm competition and harm consumers.

Proponents of the view that loyalty discounts should be analyzed under a rule-of-reason analysis similar to exclusive dealing cases point to this possibility as the basis for their view. However, a model of competition implied by the discount attribution suggests the concerns that give rise to the possible need to conduct a rule of reason analysis may not be important when sellers compete for sales to intermediaries that have both contested and non-contested purchases because the direct effect of loyalty discounts benefits consumers. The analysis in this paper shows that making it harder for less preferred sellers to compete increases competition and directly benefits buyers. The indirect effect of allowing sellers to offer loyalty discounts could be to exclude competitors and increase monopoly power if there are scale economies. However, allowing a more preferred seller to offer loyalty discounts has the same effect as a reduction in its production costs, and sellers typically are not subject to antitrust scrutiny as a result of lowering their production costs even if doing so could indirectly lead to monopoly power. More generally, sellers are not subject to antitrust scrutiny for engaging in conduct that directly increases competition and benefits buyers. The one exception to this is when sellers reduce prices sufficiently that they are potentially predatory.

Finally, the discount attribution tests can be used to assess whether loyalty discounts are being used to exclude equally efficient rivals, passing the discount attribution test is not sufficient for a seller’s loyalty discounts to benefit buyers. If a more preferred seller that has costs sufficient below its rival’s costs would make contested sales in a single-price equilibrium and in a loyalty discount equilibrium, then it could use loyalty discounts to reduce the less preferred seller’s constraint on its prices. In this case, the harm to buyers is not due to harm to rivals, but, instead, results from the fact that loyalty discounts are a form of price discrimination.
Nonetheless, this analysis suggests a simple test of whether a more preferred seller offering loyalty discounts would make contested sales even without loyalty discounts. Since the more preferred seller would charge its rival’s profit-neutral price in single price equilibrium, if we observe a seller offering loyalty discounts with discounted prices that are above its rival’s profit-neutral price, then the seller’s price is higher with loyalty discounts than they would be in an equilibrium without loyalty discounts. For example, suppose we observe seller A offering a loyalty discount price of 7. If using (2), seller B’s profit-neutral price is 6.67, this tells us that buyer costs are higher with loyalty discounts.
APPENDIX

1. If sellers are equally efficient, the more preferred seller has the higher profit-neutral price

Seller i’s profit from selling non-contested units only is $Q_i^n (P_i^n - C_i)$. If it offers a price $P_i^*$ below $P_i^n$ in order to win contested sales, its profit from selling both contested and non-contested units is $(Q_i^n + Q^c)(P_i^* - C_i)$. Setting these two expressions equal to each other yields:

$$ (1) \quad P_i^* = C_i + \frac{Q_i^n (P_i^n - C_i)}{(Q_i^n + Q^c)} = \frac{P_i^n Q_i^n + Q^c C_i}{(Q_i^n + Q^c)} $$

$P_i^*$ is increasing in both $Q_i^n$ and $P_i^n$. As a result, if $C_a = C_b$, $Q_a^n > Q_b^n$, and $Q_a^n P_a^n > Q_b^n P_b^n$, then $P_a^* > P_b^*$.

2. The lower production cost seller wins contested sales in the loyalty-discount equilibrium

If each seller offers the buyer $P_i^*$ if it buys all its contested units from it and $P_i^n$ if it does not, then the buyer’s cost, if it accepts A’s loyalty discounts offer, is $P_a^*(Q_a^n + Q^c) + Q_b^n P_b^n$. Using (1), this simplifies to $Q_a^n P_a^n + Q^c C_a + Q_b^n P_b^n$. Doing the same for seller B, yields $Q_a^n P_a^n + Q^c C_b + Q_b^n P_b^n$. Thus, the buyer will purchase contested units in the loyalty-discount equilibrium from the seller with the lower $C_i$. If $C_a = C_b$, the buyer will be indifferent between the two loyalty discount offers.

3. Buyer costs are lower in the loyalty-discount equilibrium if the less preferred seller loses sales as a result of loyalty discounts

Since the lower production cost seller wins sales in the loyalty-discount equilibrium, seller B loses sales because of loyalty discounts if $C_a \leq C_b$ and $P_a^* > P_b^*$. If $P_a^* > P_b^*$, then B can charge a price just below $P_a^*$ and can profitably make all the contested sales. Thus, buyer costs in the single-price equilibrium equal:

$$ (2) \quad P_a^*(Q_b^n + Q^c) + Q_a^n P_a^n $$

The profit maximizing price for the lower production cost seller in the loyalty-discount equilibrium sets buyer costs from its loyalty discount offer equal to the buyer cost if its rival charged its profit-neutral price. If seller B offers a loyalty discount equal to $P_b^n$, then the buyer
costs if it accepted that offer would be \(P_a^*(Q_b^n + Q^c) + Q_a^n P_a^n\). Let \(\hat{P}_a\) be the profit maximizing price for seller A. That price satisfies:

\[
(3) \quad \hat{P}_a(Q_a^n + Q^c) + Q_b^n P_b^n = P_a^*(Q_b^n + Q^c) + Q_a^n P_a^n
\]

Solving for \(\hat{P}_a\) yields:

\[
(4) \quad \hat{P}_a = \frac{Q_a^n P_a^n + Q^c C_b}{Q_a^n + Q^c}
\]

Note that if \(C_a = C_b\) then according to (4) and (1) \(\hat{P}_a = P_a^*\), so sellers charge their profit-neutral prices in the loyalty-discount equilibrium if they have the same costs.

Inserting (4) into the left hand side of (3) yields buyer costs in the loyalty-discount equilibrium of:

\[
(5) \quad Q_a^n P_a^n + Q^c C_b + Q_a^n P_a^n
\]

Using (2), one can rewrite the expression of buyer costs in the single-price equilibrium as:

\[
(6) \quad \alpha Q_a^n P_a^n + \alpha Q^c C_b + Q_a^n P_a^n
\]

where \(\alpha = P_a^*/P_b^* > 1\). Comparing (5) and (6) reveals that buyer costs are higher in the single-price equilibrium.

4. **The effect of loyalty discounts on buyer costs is ambiguous when the same seller makes contested sales in the single-price equilibrium and the loyalty-discount equilibrium.**

For seller B to make contested sales in the LDE, it must have lower production costs. According to (1), \(P_i^*\) is a decreasing function of \(C_i\), so if \(C_b < C_a\) and B is the less preferred seller, \(P_b^* < P_a^*\). This means that if seller A has higher costs, seller B makes contested sales in both the single-price equilibrium and the loyalty-discount equilibrium.

If seller B makes contested sales in both settings, then buyers pay \(P_a^n\) for \(Q_a^n\) with and without loyalty discounts. For \(Q^c\) and \(Q_b^n\), buyers pay \(P_a^*\) in the single-price equilibrium and \(\hat{P}_b\) in the loyalty-discount equilibrium. Using (1) and (4), buyers are better off with loyalty discounts if:

\[
(7) \quad \frac{Q_b^n P_b^n}{(Q_b^n + Q^c)} < \frac{Q_a^n P_a^n}{(Q_a^n + Q^c)}
\]
Because A is the more preferred seller, this condition is necessarily satisfied.

If we drop the assumption that $Q_a^n P_a^n > Q_b^n P_b^n$, then (7) may not be satisfied even if $Q_a^n > Q_b^n$. In other words, if the non-contested price for B is sufficiently high relative to the non-contested price for A, then seller B may be able to charge a higher price in the loyalty-discount equilibrium.

For seller A to make contested sales in both the single-price equilibrium and the loyalty-discount equilibrium, its costs must be sufficiently less than B’s that $P_a^* < P_b^*$ even though A is the more preferred seller. If A makes contested sales with and without loyalty discounts, then buyers pay $P_b^n$ for $Q_b^n$. For $Q^c$ and $Q_b^n$, buyers pay $P_b^*$ in the single-price equilibrium and $\hat{P}_a$ in the loyalty-discount equilibrium. Using (1) and (4), buyers are better off with loyalty discounts if:

$$\frac{Q_a^n P_a^n}{(Q_b^n + Q^c)} > \frac{Q_a^n P_a^n}{(Q_a^n + Q^c)}$$

Because A is the more preferred seller, the right hand side exceeds the left hand side, and buyer costs are higher with loyalty discounts. If we drop the assumption that $Q_a^n P_a^n > Q_b^n P_b^n$, then (8) may be satisfied even if $Q_a^n > Q_b^n$. 
